

## THE SPECTRUM OF $q$ -CESÀRO MATRICES ON $c$ AND ITS VARIOUS SPECTRAL DECOMPOSITION FOR $0 < q < 1$

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*Abstract.* One of  $q$ -analogs of the Cesàro matrix of order one is the triangular matrix with nonzero entries  $c_{nk} = \frac{q^{n-k}}{1+q+\dots+q^n}$ ,  $0 \leq k \leq n$ , where  $q \in [0, 1]$ . In this article, we will determine the spectrum of this matrix on the space of convergent sequences  $c$ . We will also obtain the fine spectral decomposition in the sense of Goldberg and a non-discrete spectral decomposition of the obtained spectrum.

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