Analyzing Period-Doubling Bifurcation and Stability for a Discrete-Time Prey- Predator Model with Allee Effect

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Abstract – Prey-predator relationship is a very important population phenomenon that occurs in nature. In population dynamics, when the population density is very low, there is a positive correlation between the population unit growth rate and the popu lation density. This phenomenon can be called the Allee effect [1], starting with Allee's research [2]. Factors such as mating difficulty, mating depression, food problem, and protection from predator are considered as Allee effect. Analysis of systems involving Allee effect has gained lots of importance in problems associated with various fields such as conservation biology [3], sustainable harvesting [4], pest control, biological control [5], population management [6], biological invasions [7], interacting species [8]. Therefore, studies on Allee effect have received more and more attention from both mathematicians and ecologists.

In this study, the qualitative behavior of a discrete time prey-predator model with Allee effect in prey population is discussed. By applying the Euler scheme method to the continuous model in [9], the discrete-time model is obtained. First, the existence of the fixed points and their topological classiffications are analyzed algebraically. Then, the conditions of existence for period-doubling bifurcation arising from coexistence fixed point with the help of center manifold theorem are investigated. Finally, numerical simulations are given to support theorical finding.

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