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ON THE PROPERTIES OF THE SET OF TRAJECTORIES OF THE NONLINEAR CONTROL SYSTEM WITH QUADRATIC INTEGRAL CONSTRAINT ON THE CONTROL FUNCTIONS

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Abstract: In this paper the control system described by a nonlinear differential equation is studied. It is assumed that the control functions have a quadratic integral constraint, more precisely, the admissible control functions are chosen from the ellipsoid of the space $L_2([t_0,\theta];\mathbb{R}^m)$. Different properties of the set of trajectories are investigated. It is proved that a small perturbation of the set of control functions causes also appropriate small perturbation of the set of trajectories. It is also shown that the set of trajectories has a small change if along with the integral constraint on the control functions, a sufficiently large norm type geometric constraint on the control functions is introduced. It is established that every trajectory is robust with respect to the fast consumption of the remaining control resource, and hence every trajectory of the system can be approximated by a trajectory generated by full consumption of the total control resource.

Keywords: Nonlinear control system, Quadratic integral constraint, Set of trajectories, Robustness.

1. Introduction

The control systems described by nonlinear differential equations are investigated in a vast number of papers. Depending on the character of the control efforts the control systems are classified as a) the control systems with geometric constraint on the control functions; b) the control systems with integral constraint on the control functions; and c) the control systems with mixed constraints on the control functions which include both the geometric and the integral constraints on the control functions. The geometric constraints on the control functions appear in the case when the control resource is not exhausted by consumption. But, if the control resource is exhausted by consumption, say as energy, food, fuel, finance, etc., then the integral constraints on the control functions is inevitable (see, e.g., [1, 2, 9, 12, 15, 16]). For example, the behaviour of the flying objects with rapidly changing mass is described as a control system with integral constraint on the control functions (see, e.g., [2, 12]).

One of the important notions of the control systems theory is the set of trajectories and attainable set concepts. Attainable set of the system at the given instant of time consists of points to which arrive the trajectories of the system and can be defined as a section of the set of trajectories at the given instant of time. Different topological properties and approximate construction methods of the set of trajectories described by various types of the integral and differential equations, where the control functions have integral constraints, are considered in papers [4–8, 11, 13, 14]. In papers [4, 5, 11, 14] the compactness, closedness, path-connectedness properties and approximate construction methods of the set of trajectories and attainable sets of the control systems which are affine with respect to the control vector are discussed. In papers [6–8, 13] the same problems are investigated for nonlinear control systems. In presented paper the properties of the set of trajecto-