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ON THE ROBUSTNESS OF CONTINUOUS TRAJECTORIES OF THE NONLINEAR CONTROL SYSTEM DESCRIBED BY AN INTEGRAL EQUATION

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Abstract. In this paper the control system described by Urysohn type integral equation is studied. It is assumed that control functions are integrally constrained. The trajectory of the system is defined as multivariable continuous function which satisfies the system's equation everywhere. It is shown that the set of trajectories is Lipschitz continuous with respect to the parameter which characterizes the bound of the control resource. An upper estimation for the diameter of the set of trajectories is obtained. The robustness of the trajectories with respect to the fast consumption of the remaining control resource is discussed. It is proved that every trajectory can be approximated by the trajectory obtained by full consumption of the control resource.

1. Introduction

The control system described by Urysohn type integral equation

$$x(\omega) = f(\omega, x(\omega)) + \int_E F(\omega, s, x(s), u(s))ds \tag{1.1}$$

is considered, where $\omega \in \Omega$, $x(\omega) \in \mathbb{R}^n$ is the state vector, $u(s) \in \mathbb{R}^m$ is the control vector, $\Omega \subset \mathbb{R}^k$, $E \subset \mathbb{R}^k$ are compact sets, $E \subseteq \Omega$.

Note that integral equations are adequate tool for description of the behavior of different processes arising in theory and applications. One of the outstanding scientist of the XX century W. Heisenberg in his well known book "Physics and Philosophy" underlines the importance of the integral equations by the following words: "The final equation of motion for matter will probably be some quantized nonlinear wave equation... This wave equation will probably be equivalent to rather

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