Research Article

Nesir Huseyin*

Approximation of the image of the L_p ball under Hilbert-Schmidt integral operator

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Abstract: In this article, an approximation of the image of the closed ball of the space L_p $(p > 1)$ centered at the origin with radius r under Hilbert-Schmidt integral operator $F(\cdot) : L_p \to L_q$, $\frac{1}{p} + \frac{1}{q} = 1$ is considered. An error evaluation for the given approximation is obtained.

Keywords: Hilbert-Schmidt integral operator, image of a set, input–output system, approximation, error evaluation

MSC 2020: 47G10, 47B38, 65R10, 93C35

1 Introduction

Integral operators arise in various problems of theory and applications and are one of the important tools to investigate different types of problems in mathematics. For example, integral operators are used in integral equations of the Fredholm, Volterra, Urysohn-Hammerstein and other types and play a crucial role in the definition of solution concepts for different types of initial and boundary value problems of differential equations (see, e.g., [1–3] and references therein). It is necessary to underline that the theory of linear integral equations is considered one of the origins of contemporary functional analysis [4–6]. In particular, the integral operators are used to describe the behavior of some input–output systems (see, e.g., [7–9]).

In this article an approximation of the image of the closed ball of the space L_p $(p > 1)$ centered at the origin under the Hilbert-Schmidt integral operator is considered. The presented approximation method allows for every $\varepsilon > 0$ to construct a finite ε-net on the image of the closed ball, which consists of the images of a finite number of piecewise-constant functions. An approximation of the image of a given closed ball can be used in infinite-dimensional optimization problems for predetermining the desirable inputs for the input–output system described by Hilbert-Schmidt integral operator. Note that the input functions with integral constraints are usually applied when the input resources of the system are exhausted by consumption, such as energy, fuel, and finance (see, e.g., [10–12] and references therein). An error evaluation of the Hausdorff distance between the image of the closed ball and its approximation, which consists of a finite number of functions, is given.

The article is organized as follows. In Section 2, the conditions and auxiliary propositions that are used in the following arguments are formulated. In Section 3, the image of the integral operator is approximated by the set, consisting of a finite number of functions. An error estimation depending on the approximation parameters is given (Theorem 1).

* **Corresponding author: Nesir Huseyin,** Department of Mathematics and Science Education, Sivas Cumhuriyet University, 58140 Sivas, Turkey, e-mail: nhuseyin@cumhuriyet.edu.tr