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ONLINE MEETING, TURKEY  
OCTOBER 22-24, 2021



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## Nonlocal Behaviours in Nature: Fractional, Fractal and Piecewise Processes

Abdon ATANGANA<sup>1</sup>

<sup>1</sup>*Institute of Ground Water Studies, Faculty of Natural and Agricultural Sciences,  
University of the Free State, Bloemfontein-SOUTH AFRICA*

[AtanganaA@ufs.ac.za](mailto:AtanganaA@ufs.ac.za)

### ABSTRACT

In the last past decades, to better understand some complex real-world behaviours, mathematicians have introduced some concepts including fractional differential and integral operators, fractal mapping and piecewise differential and integral operators. The aim of this talk is to present theories and applications of these new trends.

## Fractional $h$ -Discrete Calculus and its Applications in Medical Sciences

**Ferhan M. ATICI<sup>1</sup>**

*<sup>1</sup>Department of Mathematics, Western Kentucky University, Bowling Green,  
Kentucky 42101-3576 USA*

[ferhan.atici@wku.edu](mailto:ferhan.atici@wku.edu)

### **ABSTRACT**

In this talk, we introduce basic definitions and some recent results in fractional  $h$ -discrete calculus. We only focus on the backward difference operators which are also known as discrete nabla operators. As an application, we study the  $h$ -discrete and  $h$ -discrete fractional representation of a pharmacokinetics-pharmacodynamics (PK-PD) model describing tumor growth and anticancer effects in continuous time considering a time scale  $h\mathbb{N}_0$ , where  $h > 0$ . Since the measurements of the drug concentration in plasma were taken hourly, we consider  $h = 1/24$  and obtain the model in discrete time (i.e. hourly). After estimating and getting confidence intervals of the model parameters, we compare residual squared sum values of the models in one table.

## Some Mathematical Problems and Their Solutions for the Oscillating Systems with Liquid Dampers (Survey)

Fikret ALIEV<sup>1</sup>, Nargiz SAFAROVA<sup>1</sup> and Nazile HAJIYEVA<sup>1</sup>

<sup>1</sup> *Institute of Applied Mathematics, BSU, Baku, Azerbaijan*

[f\\_aliev@yahoo.com](mailto:f_aliev@yahoo.com)  
[narchis2003@yahoo.com](mailto:narchis2003@yahoo.com)  
[nazile.m@mail.ru](mailto:nazile.m@mail.ru)

### ABSTRACT

The mathematical problem of an oscillating system with liquid dampers is considered, such as finding the order of the fractional derivative of a subordinate term based on the given statistical data from practice, constructing a solution of the corresponding system with nonseparated boundary conditions, including for large values of the head mass, finding asymptotic solutions on the first approximations, and constructing optimal regulators to stabilize the system around the corresponding program trajectories and controls [1-4].

For the first time, an inverse problem of the third generation is presented for determining the order of the derivative of the subordinate term of the differential equation of oscillatory systems with liquid dampers (OSLD). Methods for solving the equation OSLD with nonlocal boundary conditions are proposed. For a sufficiently large mass, an asymptotic method is constructed. It is noted that this method can be useful for constructing programmed trajectories and controls for oscillatory systems with liquid dampers. An algorithm for constructing optimal controllers with the Letov time method and Larin's frequency parameterization method is given. An asymptotic method is also presented in the first approximation of constructing controllers. Finally, a method for discretizing the OSLD is proposed, which, in contrast to the classical case, has nonstationary linear equations.

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### REFERENCES

- [1] F.A. Aliev, N.A. Aliev, N.A. Safarova, *Appl. Comput. Math.*, 18(3), 316-325 (2019).
- [2] F.A. Aliev, N.A. Aliev, M.M. Mutallimov, A.A. Namazov, *Appl. Comput. Math.*, 19(3), 415-422 (2020).
- [3] B. Bonilla, M. Rivero, J.J. Trujillo, *Appl. Math. Comput.*, 187, 68-78 (2007).
- [4] F.A. Aliev, N.A. Aliev and N.A. Safarova, et al., *J. Appl. Comput. Mech.*, 6(SI), 1426-1430 (2020).

## Machine-Deep Learning and Finance: a Review of Recent Results

Luca GRILLI<sup>1</sup> and Domenico SANTORO<sup>2</sup>

<sup>1</sup>*Department of Economics, Management and Territory,  
Università degli Studi di Foggia, Foggia-ITALIA*

<sup>2</sup>*Department of Economics and Finance,  
Università degli Studi di Bari, Bari-ITALIA*

[luca.grilli@unifg.it](mailto:luca.grilli@unifg.it)

### ABSTRACT

In the last few years the application of Machine/Deep (ML/DL) Learning in finance has faced increasing interest. In this talk, we are going to introduce some recent results in this field of application derived from our recent research. In particular, we present different approaches in the applications of ML/DL in finance. In the first part, we consider some stocks traded in financial markets and we define the Boltzmann Entropy in a model describing the behavior of financial markets and we show how this indicator can be used to improve the price forecasting with a neural network based on Long-Short Term Memory (LSTM) architecture. In addition, we start from the Ergodic Theory, we try to present a model describing the dynamics of the Bitcoin cryptocurrency system. We show that the Bitcoin dynamics appears as dual: it mostly behaves as a deterministic system and in some time intervals, much shorter, it enters a stochastic regime. We also try to identify patterns in this "phase transition".

At the same time, we consider two types of financial instruments traded on the market: stocks and cryptocurrencies. Stocks are traded in a market subject to opening and closing hours, whereas cryptocurrencies are traded in a 24/7 market. Herein, we employ a type of Generative Adversarial Network (GAN) to demonstrate that different amounts of information can be obtained based on the prices (and returns) of these financial instruments. We demonstrate using TimeGAN that the prices of cryptocurrencies present higher discriminatory and predictive power than stocks. In addition, we show that some stocks have the same discriminative and predictive power as cryptocurrencies.

At the end of the talk, we present an application of Natural Language Processing (NLP) to finance. BERT (Bidirectional Encoder Representations from Transformers) is one of the most popular techniques in Natural Language Processing (NLP) for Sentiment Analysis. The main goal is to classify sentences (or entire texts) and to obtain a score in relation to their polarity: positive, negative or neutral. Recently, a Transformer-based architecture, the fine-tuned ALBERTo, has been introduced in order to determine a sentiment score in the financial sector, through a specialized corpus of phrases. We use the sentiment (polarity) score to improve the forecast of stocks. We apply the BERT model to determine the score associated with various events (both positive and negative) that have affected some stocks in the market. The phrases used to determine the scores are newspaper articles published on MilanoFinanza. We compute both the average sentiment score and the polarity, and we use a Monte Carlo method to generate (starting from the day the article was released) a series of possible paths for the next 5 trading days (45 trading hours), exploiting the Bayesian inference to determine a new series of bounded drift and volatility values on the basis of the score; returning an exact "directed" price.

## REFERENCES

- [1] L. Grilli, D. Santoro, *Chaotic Modeling and Simulation*, 2021(2), 91-103 (2021).
- [2] L. Grilli, D. Santoro, *Applied Mathematical Sciences*, 15(7), 297-320 (2021).
- [3] L. Grilli, D. Santoro, *Generative Adversarial Network for Market Hourly Discrimination*, Proceedings Book of ICMRS 2020 - 3rd International Conference On Mathematical and Related Sciences: Current Trends And Developments, Editors Erhan Set, Ahmet Ocak Akdemir, Alper Ekinici, 2021, ISBN: 978-625-409-146-9, pp. 106-113.
- [4] L. Grilli, D. Santoro, *A Statistical Ensemble Based Approach for Entropy in Cryptocurrencies Markets*, CHAOS 2020 Proceedings 13th Chaotic Modeling and Simulation International Conference, edited by Christos H. Skiadas, 2020, pp. 265-277.



## Sharp regularity theorems for Minimizers of variational integrals

Maria Alessandra RAGUSA<sup>1</sup>

<sup>1</sup>*Department of Mathematics, University of Catania, Catania-ITALY*

[mariaalessandra.ragusa@unict.it](mailto:mariaalessandra.ragusa@unict.it)

### ABSTRACT

Are proved, in cooperation with prof. Atsushi Tachikawa, sharp regularity theorems for minimizers

$$u(x): \Omega \subset \mathbb{R}^m \rightarrow \mathbb{R}^n$$

of some class of variational integrals

$$\int_{\Omega} A(x, u, Du) dx$$

where  $\Omega$  is a bounded domain in  $\mathbb{R}^m$ . Concerning the dependence of integrand on the variable  $x$  is assumed only that  $A(\cdot, u, p)$  is in the vanishing mean oscillation class. Namely, is not assumed the continuity of  $A(x, u, p)$  with respect to  $x$ .

Are considered both partial and global regularity of the minimizers  $u$ .

## Some Recent Developments on Discrete Fractional Calculus

Thabet ABDELJAWAD<sup>1,2</sup>

<sup>1</sup>*Department of Mathematics and General Sciences,  
Prince Sultan University, Riyadh 11586, SAUDI ARABIA*

<sup>2</sup>*China Medical University Hospital, China Medical University, Taichung 40402, TAIWAN*

[tabdeljawad@psu.edu.sa](mailto:tabdeljawad@psu.edu.sa)

### ABSTRACT

Discrete fractional operators including fractional sums (discrete fractional integrals) and fractional differences (discrete fractional derivatives) are the discrete counterparts of fractional operators on the time scale  $\mathbb{Z}$  or more generally on  $h\mathbb{Z}$ ;  $h > 0$ . In this talk, I try to review some of the recent developments on the theory and applications of discrete fractional operators with different kernels. Mainly, two classes of fractional difference operators will be outlined. The first type is based on the iteration of the (delta or nabla) summation to produce the fractional differences with power law kernels. The second part is based on the discretization of fractional operators with exponential and Mittag-Leffer kernels.

## The convex combination: a powerful tool in the construction of convergent fixed point iterative algorithms

Vasile BERINDE<sup>1,2</sup>

<sup>1</sup>*Department of Mathematics and Computer Science, Faculty of Sciences, Technical University of Cluj-Napoca North University Center at Baia Mare  
Baia Mare, ROMANIA*

<sup>2</sup>*Academy of Romanian Scientists, Bucharest, ROMANIA*

[vasile.berinde@mi.utcluj.ro](mailto:vasile.berinde@mi.utcluj.ro)

[vasile.berinde@gmail.com](mailto:vasile.berinde@gmail.com)

### ABSTRACT

Our aim in this paper is to highlight the merits of a simple and natural geometrical concept – the convex combination – in nonlinear analysis and more specifically in fixed point theory. Some very recent results on this topic are surveyed [1-10] and various future related directions of research are also indicated.

### REFERENCES

- [1] V. Berinde, Carpathian J. Math., 35(3), 293-304 (2019).
- [2] V. Berinde, Carpathian J. Math., 36(1), 27-34 (2020).
- [3] V. Berinde, M. Păcurar, J. Comput. Appl. Math., 386, Paper No. 113217, 1-9 (2021).
- [4] V. Berinde, M. Păcurar, J. Fixed Point Theory Appl. 22(2), Paper No. 38, 1-10 (2020).
- [5] V. Berinde, M. Păcurar, Carpathian J. Math. 37(2), 173-184 (2021).
- [6] V. Berinde, M. Păcurar, J. Fixed Point Theory Appl. 23, 66 (2021).
- [7] V. Berinde, M. Păcurar, Symmetry 13, Article Number 713, (2021).
- [8] V. Berinde, M. Păcurar, Symmetry 13, Article Number 498, (2021).
- [9] V. Berinde, M. Păcurar, J. Fixed Point Theory Appl. 23, Art. Number 66, (2021).
- [10] M. Abbas, R. Anjum, V. Berinde, Symmetry. 13(8), Art. Number 1350, (2021).

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## Homogeneous q-Difference Equations for q-Polynomials and Some Applications

Jian CAO<sup>1</sup>

<sup>1</sup>*Department of Mathematics, Hangzhou Normal University, CHINA*

[21caojian@hznu.edu.cn](mailto:21caojian@hznu.edu.cn)

### ABSTRACT

In this talk, our aim is to build generalized homogeneous q-difference equations for q-polynomials. We also consider their applications to generating functions and fractional q-integrals by using the perspective of q-difference equations. In addition, we also reveal relevant relations of various special cases of our main results involving some known results.

## The Influence of c-subnormality of Subgroups on the Structure of Finite Groups

Jehad Al JARADEN<sup>1</sup>

<sup>1</sup>*Department of Mathematics, Al-Hussein Bin Talal University, Ma'an, JORDAN*

[jjaraden@mtu.edu](mailto:jjaraden@mtu.edu)

### ABSTRACT

Let  $H$  be a subgroup of a group  $G$ . We say that  $H$  is c-subnormal in  $G$  if there exists a subnormal subgroup  $T$  of  $G$  such that  $HT = G$  and  $H \cap T \subseteq H_G$ . In this work we shall investigate the influence of c-subnormality of some subgroups on the structure of finite groups further, and obtain some results on some kinds of weaker conditions.

## Analyzing a West Nile Virus Model in the Sense of Atangana-Baleanu Fractional Operator

Mustafa Ali DOKUYUCU<sup>1</sup>, Hemen DUTTA<sup>2</sup>, Ercan ÇELİK<sup>3</sup>

<sup>1</sup>*Department of Mathematics, Faculty of Science and Arts,  
Ağrı İbrahim Çeçen University, Ağrı-TURKEY*

<sup>2</sup>*Department of Mathematics, Gauhati University, Guwahati, 781014, INDIA*

<sup>3</sup>*Department of Mathematics, Faculty of Science,  
Atatürk University, Erzurum-TURKEY*

[mustafaalidokuyucu@gmail.com](mailto:mustafaalidokuyucu@gmail.com)

[hemen\\_dutta08@rediffmail.com](mailto:hemen_dutta08@rediffmail.com)

[ercelik@atauni.edu.tr](mailto:ercelik@atauni.edu.tr)

### ABSTRACT

In this study, the west nile virus model was analyzed with the Atangana-Baleanu fractional derivative operator. During this analysis, the existence and uniqueness solutions of the mathematical model were investigated. Then, numerical solutions were calculated using numerical methods and their simulations were drawn.

### REFERENCES

- [1] A. Atangana and D. Baleanu, *Thermal Science*, 20(2), 763-779 (2016).
- [2] J. Jiang, Z. Qiu, *SIAM Journal on Applied Mathematics*. 69(5), 1205-1227 (2009).
- [3] I. Podlubni, *Fractional Differential Equations*, Academic Press, San Diego, 1999.

## Investigating the Performance of Almost Unbiased Estimators in Gamma Regression Model

Merve KORKMAZ<sup>1</sup> and Yasin ASAR<sup>2</sup>

<sup>1</sup>*Necmettin Erbakan University, Graduate School of Natural and Applied Sciences,  
Konya, TURKEY*

<sup>2</sup>*Necmettin Erbakan University, Department of Mathematics and Computer Science,  
Konya, TURKEY*

[krkmzmerve70@gmail.com](mailto:krkmzmerve70@gmail.com)

### ABSTRACT

Generalized linear models firstly proposed by Nelder and Wedderburn (1972) can be used to model both discrete and continuously dependent variables without assumptions of normality and constant variance, such as linear models (Khuri, 2010). Gamma regression model can be used when the distribution of the response variable is gamma distribution, which is belonging to the exponential family of distributions. The maximum likelihood (ML) method is commonly used to estimate model parameters in the gamma regression model. The ill-conditioning problem (multicollinearity) causes a high variance in the likelihood estimates, so the standard errors of the ML estimator are high and unstable. Also, ML estimators can have wrong signs. In the literature, some biased estimators have been used as a solution to the multicollinearity problem in gamma regression. Almost unbiased estimators, on the other hand, can also be used when there is multicollinearity. In this study, Liu type estimator and almost unbiased Liu type estimator are examined in case of ill-conditioning in the gamma regression model. A Monte Carlo simulation is designed, and the performances of the estimators are compared according to the mean squared error and squared bias criteria. The almost unbiased Liu type estimator performed better than the maximum likelihood estimator and the Liu type gamma estimator according to the simulation results.

### REFERENCES

- [1] A.I. Khuri, *Linear model methodology*, FL Chapman & Hall/CRC Press, 2010.
- [2] J.A. Nelder, R.W. Wedderburn, *Journal of the Royal Statistical Society: Series A (General)*, 135(3), 370-384 (1972).

## On Slant Lightlike Submersions

Ramazan SARI<sup>1</sup>

<sup>1</sup>*Gümüřhacıköy Hasan Duman Vocational School, Amasya University, Amasya-TURKEY*

[ramazan.sari@amasya.edu.tr](mailto:ramazan.sari@amasya.edu.tr)

### ABSTRACT

In this paper, We defined and studied slant lightlike submersions from indefinite Sasakian manifold onto a lightlike manifold. We investigated the geometry of foliations which arise from the definition of this new submersion. We obtained necessary and sufficient condition for base manifold to be a locally product manifold.

### REFERENCES

- [1] B. O'Neill, Mich. Math. J., 13, 459-469 (1966).
- [2] B. O'Neill, *Semi-Riemannian Geometry with applications to Relativity*, Academic press, New York-London, 1983.
- [3] R. Prasad, P. Kumar Singh and S. Kumar, J. Math. Comput. Sci, 8, 324-349 (2018).
- [4] R. Sachdeva, R. Kumar, S. S. Bhatia, Ukrainian Mathematical Journal, 68(7), 225-240 (2016).
- [5] B. Şahin, Mediterr. J. Math., 5, 273-284 (2008).
- [6] B. Şahin, Bull. Math. Soc. Sci. Math. Roumanie, 54, 102, 93-105 (2011).
- [7] B. Şahin and Y. Gündüzalp, Hacet. J. Math. Stat., 39,41-53 (2010).



## The Quaternionic Darboux Ruled Surface in terms of Bishop Frame

Abdussamet ÇALIŞKAN<sup>1</sup>

<sup>1</sup>*Accounting and Tax Applications, Fatsa Vocational School,  
Ordu University, Ordu-TURKEY*

[abdussamet65@gmail.com](mailto:abdussamet65@gmail.com)

### ABSTRACT

In this paper, we investigate the quaternionic expression of the ruled surface drawn by the motion of the Bishop Darboux vector. The distribution parameters, the pitches, and the angle of pitches of the ruled surface are calculated as quaternionic.

### REFERENCES

- [1] K. Bharathi and M. Nagaraj, *Ind. J. P. Appl. Math.*, 18, 507-511 (1987).
- [2] L. R Bishop, *The American Mathematical Monthly*, 82(3), 246-251 (1975).
- [3] W. R. Hamilton, *Elements of quaternions*, Longmans, Green Company, 1866.
- [4] M. Masal, A.Z. Azak, *Proceedings of the National Academy of Sciences, India Section A: Physical Sciences*, 89(2), 415-424 (2019).
- [5] Y. Tunçer, *Gen Math Notes*, 26, 74-83 (2015).

## Existence and Uniqueness of The Weak Solution for Keller-Segel Model Coupled With The Heat Equation

Ali SLIMANI<sup>1</sup> and Amar GUESMIA<sup>1</sup>

<sup>1</sup>*Department of Mathematics, University 20 august 1955, ALGERIA,  
Laboratory applied mathematics and history and Didactics of mathematics*

[alislmani21math@gmail.com](mailto:alislmani21math@gmail.com), [ali.slimani@univ-skikda.dz](mailto:ali.slimani@univ-skikda.dz)  
[guesmiaamar19@gmail.com](mailto:guesmiaamar19@gmail.com)

### ABSTRACT

Keller-Segel chemotaxis model is described by a system of nonlinear PDE : a convection diffusion equation for the cells density coupled with a reaction-diffusion equation for chemoattractant concentration. In this work, we study the phenomenon of Keller Segel model coupled with a heat equation, because The heat has an effect the density of the cells as well as the signal of chemical concentration, since the heat is a factor affecting the spread and attraction of cells as well in relation to the signal of chemical concentration, The main objectives of this work is the study of the global existence and uniqueness and boundedness of the weak solution for the problem defined in (2.4) for this we use the technical of Galerkin method.

### ACKNOWLEDGEMENT

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### REFERENCES

- [1] A. Slimani, A. Rahai and A. Guesmia, L. Bouzettouta, Int. J. Anal. Appl., 19(0), 1-32 (2021), DOI: 10.28924/2291-8639-19-2021-1.
- [2] E.O. Budrene, H.C. Berg, Nature (London), 349, 630-633 (1991).
- [3] A. Lorz, Communications in Mathematical Sciences, 10(2), 555-574 (2012).
- [4] A. Guesmiai N. Daili, Acta Universitatis Apulensis 21, 161-170 (2010).

## On the Weighted Generalized $q$ -Hermite-Hadamard inequalities

Necmettin ALP<sup>1</sup>

<sup>1</sup>*Department of Mathematics, Faculty of Science and Arts,  
Düzce University, Düzce-TURKEY*

[placenn@gmail.com](mailto:placenn@gmail.com)

### ABSTRACT

The aim of this work is to prove quantum estimates for weighted generalized  $q$ -Hermite-Hadamard type inetgral inequalities by using two kind  $q$ -integral definitions. It is shown that the classical results can be obtained by taking limit of the inequalities presented in this study as  $q \rightarrow 1^-$ .

### REFERENCES

- [1] R. Agarwal, Comptes rendus de l'Academie des Sciences, 236(21), 2031-2032 (1953).
- [2] W. Al-Salam, Proceedings of the Edinburgh Mathematical Society, 15(2), 135-140 (1966/1967).
- [3] N. Alp, M. Z. Sarikaya, M. Kunt and I. Iscan, Journal of King Saud University-Science 30, 193-203 (2018).
- [4] S. Bermudo, P. Kórus, and JE Nápoles Valdés, Acta mathematica hungarica (2020): 1-11.
- [5] T. Ernst, *The History of  $Q$ -Calculus And New Method*. Sweden: Department of Mathematics, Uppsala University, 2000.
- [6] F. H. Jackson, Quarterly J. Pure Appl. Math. 41, 193-203 (1910).
- [7] V. Kac and P. Cheung *Quantum calculus*, Springer, 2001.
- [8] P. M. Rajkovic, M. S. Stankovic, and S. D. Marinkovic, *The zeros of polynomials orthogonal with respect to  $q$ -integral on several intervals in the complex plane*, Proceedings of The Fifth International Conference on Geometry, Integrability and Quantization, pp. 178-188, 2003.
- [9] W.U, Shanhe. The Rocky Mountain Journal of Mathematics, 2009, 1741-1749.
- [10] J. Tariboon, S. K. Ntouyas, Adv. Difference Equ. 282, 1-19 (2013).

## Algorithmic Approach on Sheffer Stroke L-algebras

Necla KIRCALI GÜRSOY<sup>1</sup>

<sup>1</sup>*Tire Kutsan Vocational School, Ege University, İzmir-TURKEY*

[necla.kircali.gursoy@ege.edu.tr](mailto:necla.kircali.gursoy@ege.edu.tr)

### ABSTRACT

We introduced a new algebraic and logical structure which is called “Sheffer stroke L-algebras” [4]. In this study, two new polynomial algorithms for Sheffer stroke L-algebras are introduced. The algorithms, which are constructed on the algebraic and logic structure, are designed to detect Sheffer stroke operation and, Sheffer stroke L-algebras, respectively.

### REFERENCES

- [1] W. Rump J. Algebra, 320, 2328-2348 (2008).
- [2] W. Rump Y.C. Yang Algebra Univ., 67(2), 121-130 (2012).
- [3] H.M. Sheffer, Transactions of the American Mathematical Society, 14(4), 481-488 (1913).
- [4] N. Kırçalı Gürsoy T. Öner A. Ülker, *On Sheffer stroke L-Algebras*, 3<sup>rd</sup> International Conference on Mathematical and Related Science: Current Trends and Developments, 20-22 November 2020, Online Meeting, Turkey.
- [5] T.H. Cormen, C.E. Leiserson, R.L. Rivest, C. Stein, *Introduction to algorithms*, MIT press, 2009.

## On the Solution of Knapsack Problem with Artificial Neural Networks

Tugay İLHAN<sup>1</sup> and Arif GÜRSOY<sup>1</sup>

<sup>1</sup>*Department of Mathematics, Faculty of Science, Ege University, İzmir-TURKEY*

[netizm@gmail.com](mailto:netizm@gmail.com)  
[arif.gursoy@ege.edu.tr](mailto:arif.gursoy@ege.edu.tr)

### ABSTRACT

One-dimensional cutting problems are combinatorial optimization problems in the NP-Hard class. The knapsack problem is one of the most widely known one-dimensional cutting problems. For these problems, there is no exact solution method in polynomial time, and it is tried to be solved with heuristic methods within reasonable time limits.

In this study, a research was carried out to solve the backpack problems with the help of artificial neural networks. By separating the problem instances into different groups according to their characteristics, appropriate artificial neural networks were selected according to the problem type. In conclusion, average efficiencies of up to 82% were obtained for the Backpack problem with the generated type-specific neural network models in this study.

### REFERENCES

- [1] T. Aktin, R.G. Özdemir, *European Journal of Operational Research*, 196(2), 737-743 (2009).
- [2] H. Dyckhoff, *European Journal of Operational Research*, 44(2), 145-159 (1990).
- [3] Ç. Elmas, *Yapay Zeka Uygulamaları*, Seçkin Yayıncılık, 2007.
- [4] J.J. Hopfield, *Proceedings of the national academy of sciences*, 79(8), 2554-2558 (1982).
- [5] B.H. Korte, J. Vygen, B. Korte, J. Vygen, *Combinatorial optimization* (Vol. 1). Berlin: Springer, 2010.
- [6] P. Toth, S. Martello, *Knapsack problems: Algorithms and computer implementations* (pp. 14-15). Chichester, UK: Wiley, 1990.
- [7] M. Minsky, S.A. Papert, *Perceptrons: An introduction to computational geometry*. MIT press, 2017.
- [8] D.E. Rumelhart, G.E. Hinton, R.J. Williams, *Nature*, 323(6088), 533-536 (1986).
- [9] D. Specht, *IEEE Transactions on Neural Networks*, 2(6), 568-576 (1991).

## Mathematical Modeling of the Width of Grooves Created with Laser Beam on Dual-Phase Steel Surface

Timur CANEL<sup>1</sup> and İrem BAĞLAN<sup>1</sup>

<sup>1</sup>*Kocaeli University, Faculty of Arts and Science, Department of Physics, 41380 Umuttepe  
Kocaeli-TURKEY*

[tcanel@kocaeli.edu.tr](mailto:tcanel@kocaeli.edu.tr)

[isakinc@kocaeli.edu.tr](mailto:isakinc@kocaeli.edu.tr)

### ABSTRACT

In this study, mathematical modeling of the width of grooves created with CO<sub>2</sub> laser on dual-phase steel has been made. Fourier and Finite difference methods were used in the mathematical model. To obtain a mathematical model, the effects of the laser power on the groove width of dual-phase steel surface were investigated. A mathematical model has been obtained by using the thermo-physical properties of dual-phase steel and laser parameters.

### ACKNOWLEDGEMENT

This work was supported by Kocaeli University Scientific Research Projects Coordination Unit (BAP, Project Number: FBA-2019-1586).

### REFERENCES

- [1] M. Nouroozi, H. Mirzadeh, M. Zamani, *Materials Science and Engineering A*, 736, 22-26 (2018).
- [2] T. Canel, M. Zeren, T. Sinmazcelik, *Optics&Laser Technology*, 120, 105714 (2019).
- [3] T. Canel, I. Baglan, T. Sinmazcelik, *Optics and Laser Technology* 115, 481-486 (2019).

## Generalized Semiderivations on Prime Rings

Evrım GÜVEN<sup>1</sup>

<sup>1</sup>*Department of Mathematics, Faculty of Science and Arts,  
Kocaeli University, Kocaeli-TURKEY*

[evrim@kocaeli.edu.tr](mailto:evrim@kocaeli.edu.tr)

### ABSTRACT

Let  $R$  be a two-torsion free prime ring. The additive map  $F$  on  $R$  is a generalized semi-derivation of  $R$ , if  $F(rs)=F(r)s+g(r)f(s)=F(r)g(s)+rf(s)$  and  $F(g(r))=g(F(r))$  for all  $r, s \in R$ . In this work, research about the investigation of generalized semi-derivations focusing on  $(\sigma, \tau)$ -Jordan ideals is presented.

### ACKNOWLEDGEMENT

This work has been supported by the Kocaeli University Scientific Research Projects Coordination Unit (ID:1599).

### REFERENCES

- [1] J. Bergen, Canadian Mathematical Bulletin, 26, 267-270 (1983).
- [2] M.V.L. Bharathi, K. Jayalakshmi, International Journal of Pure and Applied Math., 113(6), 101-109 (2017).
- [3] M. Bresar, Glasgow Math. 33, 89-93(1991).
- [4] J.C. Chang, Chinese Journal of Math., 12, 255-262 (1984).
- [5] V. De Filippis, A. Mamouni, L. Oukhtite, Canadian Mathematical Bulletin, 58(2), 263-270 (2015).
- [6] A. Fırat, International Journal of Pure and Applied Mathematics, 28(3), 363-368 (2006).
- [7] E. Güven, East-West J. Mathematics, 21(1), 58-69 (2019).
- [8] E. Güven, TWMS J. App. and Eng. Math., 9(1), 22-29 (2019).
- [9] E. Güven, K. Kaya, M. Soytürk, Okayama Math. J., 49, 59-64 (2007).
- [10] C. Haetinger, A. Mamouni, Palestine Journal Math., 7, 28-35 (2018).
- [11] K. Kaya, H. Kandamar N. Aydın, Doga-Tr. J. Math., 17, 251-258 (1993).
- [12] K. Kaya, Ö. Gölbaşı, N. Aydın, App. Math. E-Notes, 1, 24-30 (2001).
- [13] J.H. Mayne, Canadian Mathematic Bulletin, 27 122-126 (1984).

## Weaker Conditions for Steffensen's Inequality and its Generalizations in Quantum Calculus Settings

Ksenija SMOLJAK KALAMIR<sup>1</sup>

<sup>1</sup>*Department of Fundamental Natural and Engineering Sciences, Faculty of Textile Technology, University of Zagreb, Zagreb-CROATIA*

[ksenija.smoljak@tff.unizg.hr](mailto:ksenija.smoljak@tff.unizg.hr)

### ABSTRACT

The classical Steffensen inequality states:

Suppose that  $f$  is decreasing and  $g$  is integrable on  $[a, b]$  with  $0 \leq g \leq 1$  and  $\lambda = \int_a^b g(t) dt$ . Then we have

$$\int_{b-\lambda}^b f(t) dt \leq \int_a^b f(t)g(t) dt \leq \int_a^{a+\lambda} f(t) dt.$$

Quantum calculus is a connection between fields of mathematics and physics, and it has played a significant role in modern mathematical analysis. Quantum calculus has developed into an interdisciplinary subject because of lots of applications.

In [4] Rajković et al. proved Steffensen's inequality in quantum calculus. In this talk we establish weaker conditions on the function  $g$  in  $q$ -Steffensen's inequality which were proved in [5]. Further, we prove weaker conditions on the function  $g$  for some generalizations of the  $q$ -Steffensen inequality obtained in [5].

### REFERENCES

- [1] H. Gauchman, *Comput. Math. Appl.*, 47, 281–300 (2004).
- [2] V. Kac, P. Cheung, *Quantum calculus*, Springer-Verlag, New York, 2002.
- [3] J. Pečarić, K. Smoljak Kalamir, S. Varošaneć, *Steffensen's and related inequalities (A comprehensive survey and recent advances)*, Monographs in inequalities 7, Element, Zagreb, 2014.
- [4] P. Rajković, M. Stanković, S. Marinković, M. Kirane, *Electron. J. Differential Equations*, 2018, Article No. 112, 1–11 (2018).
- [5] K. Smoljak Kalamir, *Mathematics*, 8(9), Article No. 1462, 1–11, (2020).
- [6] J. F. Steffensen, *On certain inequalities between mean values and their application to actuarial problems*, *Skand. Aktuarietids.*, 82–97, 1918.
- [7] S.-H. Wu, H. M. Srivastava, *Appl. Math. Comput.* 192(2), 422–428 (2007).



## Finite Domination Type for Monoids

Anjeza KRAKULLI<sup>1</sup> and Elton PASKU<sup>2</sup>

<sup>1</sup>*Department of Mathematics, Faculty of Technology and Information,  
Aleksander Moisiu University, Durres-ALBANIA*

<sup>2</sup>*Department of Mathematics, Faculty of Natural Sciences,  
Tirana University, Tirana-ALBANIA*

[anjeza.krakulli@gmail.com](mailto:anjeza.krakulli@gmail.com)

[elton.pasku@fshn.edu.al](mailto:elton.pasku@fshn.edu.al)

### ABSTRACT

In [4], Squier, Otto and Kobayashi explored a property for monoids called finite derivation type (FDT for short) in an attempt to distinguish between monoids which are given by a finite convergent rewriting system and those satisfying the homological property  $FP_n$  for  $n \geq 1$ . It was realized latter in [2] that the property  $FP_1$  was related with another property called in [2] finite domination type rather than it was with FDT. In this paper we extend the notion of finite domination for rewriting systems in such a way that it generalizes FDT and likewise FDT it is an invariant of the monoid presentation. Also our finite domination type property has a stronger homotopical flavor than Kobayashi's condition has, which gives points to the idea that it might be used to relate the monoid presentation with stronger finiteness conditions than the property  $FP_1$ .

### REFERENCES

- [1] J. Isbell, Amer. J. Math. 90, 1025-1030 (1968).
- [2] Y. Kobayashi, Internat. J. Algebra Comput., 17(3), 593-605 (2007).
- [3] Y. Lafon, JPAA, 98, 229-244 (1995).
- [4] C.C. Squier, F. Otto, Y. Kobayashi, Theoret. Comput. Sci., 131, 271-294 (1994).
- [5] R. Street, *Categorical Structures*, Handbook of Algebra, Vol. 1, Elsevier Sciences, 1996.
- [6] X. Wang, S.J. Pride, Int. Journal of Algebra and Computation, 10, 425-456 (2000).

## **Solution of Higher Order Quasi-Linear Parabolic Equation Subject to Periodic Boundary Conditions Using Fourier Method**

**İrem BAĞLAN<sup>1</sup> and Timur CANEL<sup>1</sup>**

*<sup>1</sup>Department of Mathematics, Faculty of Science and Arts,  
Kocaeli University, Kocaeli-TURKEY*

[isakinc@kocaeli.edu.tr](mailto:isakinc@kocaeli.edu.tr)

### **ABSTRACT**

In this work, higher order inverse quasi-linear parabolic problem was investigated. It demonstrated the solution by the Fourier approximation. It proved the existence, uniqueness of the solution by Fourier and iteration method.

### **ACKNOWLEDGEMENT**

This work was supported by BAP (The Scientific Research Projects Coordination Unit in Kocaeli University).

### **REFERENCES**

- [1] M.Deaghan, Math. And Computers in Simulation, 49, 331-349 (1999).
- [2] M.Deaghan, Math. Comput. Model, 41, 197-213 (2005).
- [3] J.R. Cannon, Y. Lin, Inverse Problems in Science and Engineering, 4,595-606 (2001).
- [4] X.Q. He, S. Kitipornchai, K.M. Liew, Journal of the Mechanics and Physics of Solids, 53, 303-326 (2005).
- [5] I. Baglan, Inverse Problems in Science and Engineering, 23(5), 884-900 (2014).

## On Kähler structures of Taub-NUT and Kerr spaces

Özgür KELEKÇİ<sup>1</sup>

<sup>1</sup>*Department of Basic Sciences & Faculty of Engineering  
University of Turkish Aeronautical Association, Ankara-TURKEY*

[okelekci@thk.edu.tr](mailto:okelekci@thk.edu.tr)

### ABSTRACT

In this paper, we study the Kählerian nature of Taub-NUT and Kerr spaces which are gravitational instanton and black hole solutions (respectively) in general relativity and as such they have an important place in gravitation research. We show that Euclidean Taub-NUT metric is hyper-Kähler with respect to the usual almost complex structures by employing an alternative explicit coframe, and Euclidean Kerr metric is locally conformally Kähler. We also show that conformally scaled Euclidean Kerr space admits a Kähler structure (Kähler metric, closed 2-form, integrable almost complex structure) by applying an appropriate conformal scaling factor.

### REFERENCES

- [1] S. W. Hawking, *Phys. Lett. A*, 60(2), 81–83 (1977).
- [2] Claude LeBrun, *Complete Ricci-flat Kähler metrics on  $C^n$  need not be flat*. Several complex variables and complex geometry, Part2, vol.52 of Proc. Sympos. Pure Math., pages 297–304. AMS (1991).
- [3] S.-T. Yau, *Open problems in geometry*, in Proceedings of Symposia in Pure Mathematics, vol. 54, 1993, pp. 1–28.
- [4] S. Dragomir and L. Ornea, *Locally conformal Kähler geometry*, vol. 155 of Progress in Mathematics. Birkhauser Boston, Inc., Boston, MA, 1998.
- [5] A.A. Babak, M. Gideon, *Complex Manifolds* 7(1), 36–61 (2020).

## Branches on Sheffer Stroke Hilbert Algebras

Tuğçe KATICAN<sup>1</sup>

<sup>1</sup>*Department of Mathematics, Faculty of Science,  
Ege University, İzmir-TURKEY*

[tugcektcn@gmail.com](mailto:tugcektcn@gmail.com)

### ABSTRACT

In this study, an atom, a branch and a chain of Sheffer stroke Hilbert algebras are introduced. We define an atom of a Sheffer stroke Hilbert algebra and show the case which a subalgebra of a Sheffer stroke Hilbert algebra is an ideal. It is illustrated that the set of all atom of a Sheffer stroke Hilbert algebra is its subalgebra but it is not an ideal. Finally, a branch and a chain on a Sheffer stroke Hilbert algebra are determined by means of its atoms and some properties are investigated. Also, we show that the element 0 is always an atom of a Sheffer stroke Hilbert algebra, and that the set of elements which are not in a branch is an ideal of this algebraic structure.

### REFERENCES

- [1] I. Chajda, Acta Universitatis Palackianae Olomucensis Facultas Rerum Naturalium Mathematica, 44(1), 19-23 (2005).
- [2] A. Diego, Ed. Hermann, Collection de Logique Math. 21, 1-54 (1966).
- [3] W.A. Dudek, B. Karamdin, S.A. Bhatti, Algebra Colloquium 18(1), 899-914 (2011).
- [4] Y. B. Jun, Math. Japon., 43, 51–54 (1996).
- [5] T. Katican, Bulletin of the International Mathematical Virtual Institute, 12(1), 41-50 (2022).
- [6] W. McCune, R. Veroff, B. Fitelson, K. Harris, A. Feist, L. Wos, Journal of Automated Reasoning, 29(1), 1-16 (2002).
- [7] T. Oner, T. Katican, A. Borumand Saeid, Categories and General Algebraic Structures with Applications, 14(1), 245-268 (2021).
- [8] T. Oner, T. Katican, A. Borumand Saeid, Journal of Intelligent and Fuzzy Systems 40(1), 759-772 (2021).
- [9] T. Oner, T. Katican, A. Borumand Saeid, Neutrosophic Sets and Systems, 42, 221-238 (2021).
- [10] H.M. Sheffer, Transactions of the American Mathematical Society, 14(4), 481-488 (1913).

## Invariant and Lacunary Invariant Statistical Convergence of Order $\eta$ for Double Set Sequences

Uğur ULUSU<sup>1</sup> and Erdinç DÜNDAR<sup>2</sup>

<sup>1</sup>*Sivas Cumhuriyet University, Sivas-TURKEY*

<sup>2</sup>*Afyon Kocatepe University, Afyonkarahisar-TURKEY*

[ugurulusu@cumhuriyet.edu.tr](mailto:ugurulusu@cumhuriyet.edu.tr)

[edundar@aku.edu.tr](mailto:edundar@aku.edu.tr)

### ABSTRACT

In this study, for double set sequences, we introduced the notions of invariant and lacunary invariant statistical convergence of order  $\eta$  ( $0 < \eta \leq 1$ ) in the Wijsman sense. Also, we investigated the inclusion relations between them.

### REFERENCES

- [1] G. Beer, Set-Valued Anal. 2, 77-94 (1994).
- [2] F. Nuray, U. Ulusu and E. Dündar, Soft Comput. 20, 2883-2888 (2016).
- [3] F. Nuray, E. Dündar and U. Ulusu, Iran. J. Math. Sci. Inform. 16, 55-64 (2021).
- [4] E. Savaş and R.F. Patterson, J. Comput. Anal. Appl. 11, 610-615 (2009).
- [5] E. Savaş, Adv. Difference Equ. 254, 10p (2013).

## Compliance Performance of Some Growth Models When They Are Logarithmic

Mehmet KORKMAZ<sup>1</sup>

<sup>1</sup>*Department of Mathematics, Faculty of Science and Arts,  
Ordu University, Ordu-TURKEY*

[mkorkmaz52@yahoo.com](mailto:mkorkmaz52@yahoo.com)

### ABSTRACT

In this study in addition to some growth models, their logarithmic growth models were investigated. In addition, the effect of these logarithmic growth models on the choice of appropriate growth model by using some model selection criteria such as coefficient of determination, error sum of squares was searched. By using data set, it is found that the results of these logarithmic growth models are better than the results of these growth models.

### REFERENCES

- [1] S. Brody, *Bioenergetics and Growth*, Rheinhold Publishing, New York, 1945.
- [2] H.E. Burkhardt and M.R. Strub, *J. Ecol.* 60, 849-873 (1974).
- [3] A.R. Kansal, S. Torquato and G.R. Harsh, *J. Theor. Biol.* 203, 367-382 (2000).
- [4] C. Kara, A. Alp and F. Can, *Turkish Journal of Fisheries and Aquatic Sciences*, 11, 367-375 (2011).
- [5] S.O. Oyamakin and A.U. Chukwu, *Journal of Applied Mathematics, Statistics and Informatics*, 11(1), 5-17 (2015).
- [6] W.E. Ricker, *Fish Physiol.* 8, 677-743 (1979).
- [7] C.P. Winsor, *Proc.Natl. Acad. Sci.* 18(1),1-8 (1932).
- [8] A. Yıldızbakan, *Analysis on mathematical models of tree growth and comparison of these models*, MSc Thesis, Institute of Natural and Applied Sciences, University of Cukurova, Turkey, (in Turkish, with abstract in English), 2005.

## Application of resurgence theory and its properties for large-N

Fatih SAY<sup>1</sup> and Burhan ALVEROĞLU<sup>2</sup>

<sup>1</sup>*Department of Mathematics, Faculty of Arts and Sciences,  
Ordu University, Ordu-TURKEY*

<sup>2</sup>*Department of Mathematics, Faculty of Engineering and Natural Sciences,  
Bursa Technical University, Bursa-TURKEY*

[fatihsay@odu.edu.tr](mailto:fatihsay@odu.edu.tr)

[burhan.alveroglu@btu.edu.tr](mailto:burhan.alveroglu@btu.edu.tr)

### ABSTRACT

It is known that conventional expansions fail to pinpoint some features of the problems in applied mathematics. One of the tools in modern asymptotic theory that handles these difficulties is resurgence asymptotic analysis [1, 2, 3] by going beyond all orders. Following pioneering works by Écalle [4], Dingle [5] and Voros [6], the theory of resurgence has attracted considerable interest in the last decades, and it now plays an important role, for example, in applied mathematics, theoretical physics, string theory, and quantum field theory [1, 7, 8, 9]. This talk presents some formal properties of the resurgence formulae, and exploits the systematic analysis of resurgence theory. By employing the resurgent analysis to a differential equation, an exponentially improved expansion within the resurgent framework will be exhibited for large-N in different sectors of the complex plane.

### REFERENCES

- [1] M. V. Berry, *Nonlinearity*, 30(6), R25 (2017).
- [2] E. Delabaere, *Lecture Notes Math*, 2155, 2016.
- [3] J. P. Boyd, *Acta Appl. Math.*, 56(1), 1-98 (1999).
- [4] J. Écalle, Tome I. Université de Paris-Sud, Département de Mathématique, 81-05, 1981.
- [5] R. B. Dingle, *Asymptotic expansions: their derivation and interpretation*, Academic Press, London-New York, 1973.
- [6] A. Voros, *Ann. Inst. Henri Poincaré*, 39, 211 (1983).
- [7] M. V. Berry, *Asymptotics, superasymptotics, hyperasymptotics...* In *asymptotics beyond all orders* (pp. 1-14). Springer, Boston, MA, 1991.
- [8] C. M. Bender and T. T. Wu, *Phys. Rev.*, D 7, 1620 (1973).
- [9] D. J. Gross and V. Periwal, *Phys. Rev. Lett.*, 60(21), 2105–2108 (1988).

## Some Results on Conformal Quasi-Hemi-Slant Riemannian Maps

Şener YANAN<sup>1</sup>

<sup>1</sup>*Department of Mathematics, Faculty of Science and Arts,  
Adiyaman University, Adiyaman-TURKEY*

[syanan@adiyaman.edu.tr](mailto:syanan@adiyaman.edu.tr)

### ABSTRACT

In this study, the notion of conformal quasi-hemi-slant Riemannian maps is defined. Some examples of conformal quasi-hemi-slant Riemannian maps are given. Lastly, geometric properties of certain distributions are examined.

### REFERENCES

- [1] B. Şahin, Riemannian Submersions, Riemannian Maps in Heritian Geometry, and Their Applications, Elsevier, London, 2017.
- [2] S. Longwap, F. Massamba, N.E. Homti, Journal of Advances in Mathematics and Computer Science 34, 1-14 (2019).
- [3] Ş. Yanan, Turkish Journal of Mathematics and Computer Science 13, 135-144 (2021).
- [4] R. Prasad, S. Kumar, S. Kumar, A. Turgut Vanlı, Gazi University Journal of Science 34, 477-491 (2021).
- [5] H.M. Taştan, B. Şahin, Ş. Yanan, Mediterranean Journal of Mathematics 13, 2171-2184 (2016).



## Smarandache Ruled Surfaces According to Bishop Frame in $E^3$

Davut CANLI<sup>1</sup>, Süleyman ŞENYURT<sup>1</sup> and Kebire Hilal AYVACI<sup>1</sup>

<sup>1</sup>*Department of Mathematics, Faculty of Science and Arts,  
Ordu University, Ordu-TURKEY*

[davutcanli@odu.edu.tr](mailto:davutcanli@odu.edu.tr)  
[senyurtsuleyman52@gmail.com](mailto:senyurtsuleyman52@gmail.com)  
[kebirehilalayvaci@odu.edu.tr](mailto:kebirehilalayvaci@odu.edu.tr)

### ABSTRACT

In the paper, some special Smarandache ruled surfaces are introduced according to Bishop frame. The characteristics of these surfaces such as developability and minimality are discussed by considering their fundamental forms and corresponding curvatures. An example for each ruled surface is also presented.

### REFERENCES

- [1] M. Masal and A. Z. Azak, Proceedings of the National Academy of Sciences, India Section A: Physical Sciences, 89(2), 415-424 (2019).
- [2] R. L. Bishop, The American Mathematical Monthly, 82(3), 246-251 (1975).
- [3] S. Ouarab, Abstract and Applied Analysis, vol. 2021, (2021).
- [4] S. Ouarab, Journal of Mathematics, vol. 2021, (2021a).
- [5] S. Ouarab, Journal of Mathematics, vol. 2021, (2021b).
- [6] A. Sarioğlugil and A. Tutar, Int. J. Contemp. Math. Sci., 2(1), (2007).

## On Some Asymptotical Deferred Equivalence Types of Order $\alpha$ for Sequences of Sets

Uğur ULUSU<sup>1</sup> and Esra GÜLLE<sup>2</sup>

<sup>1</sup>*Sivas Cumhuriyet University, Sivas-TURKEY*

<sup>2</sup>*Afyon Kocatepe University, Afyonkarahisar-TURKEY*

[ugurulusu@cumhuriyet.edu.tr](mailto:ugurulusu@cumhuriyet.edu.tr)

[egulle@aku.edu.tr](mailto:egulle@aku.edu.tr)

### ABSTRACT

In this study, we introduced the notions of asymptotical deferred invariant, strongly deferred invariant and deferred invariant statistical equivalence of order  $\alpha$  ( $0 < \alpha \leq 1$ ) in the Wijsman sense for sequences of sets. Also, we investigated some properties of these notions and gave a relation between them.

### REFERENCES

- [1] R. P. Agnew, Ann. of Math. 33, 413-421 (1932).
- [2] G. Beer, Set-Valued Anal. 2, 77-94 (1994).
- [3] M. Et, H. Altınok and R. Çolak, AIP Conf. Proc. 1926, 020016 (2018)
- [4] M. Küçükaslan and M. Yılmaztürk, Kyungpook Math. J. 56, 357-366 (2016).
- [5] F. Nuray, J. Comput. Sci. & Comput. Math. 10, 1-6 (2020).

## A Numerical Investigation of the Eigenvalues of a Class of Non-self-adjoint Random Matrices

Hasen Mekki ÖZTÜRK<sup>1</sup>

<sup>1</sup>*Department of Mathematics, Faculty of Science and Arts,  
Ordu University, Ordu-TURKEY*

[hasenozturk@odu.edu.tr](mailto:hasenozturk@odu.edu.tr)

### ABSTRACT

This research focuses on a class of tridiagonal non-self-adjoint matrices which arise from a sign-indefinite, self-adjoint, linear pencils of matrices. In particular, we discuss the following problem. Fix an integer  $n \in \mathbb{N}$ , and define the  $n \times n$  classes of matrices

$$H_c = \begin{pmatrix} c & 1 & & & \\ 1 & c & 1 & & \\ & \ddots & \ddots & \ddots & \\ & & 1 & c & 1 \\ & & & 1 & c \end{pmatrix}, \quad D_{\mp} = \begin{pmatrix} \mp 1 & & & & \\ & \mp 1 & & & \\ & & \ddots & & \\ & & & \ddots & \\ & & & & \mp 1 \end{pmatrix},$$

where  $c \in \mathbb{R}$  is a parameter and the entries of the diagonal matrix  $D_{\mp}$  are independent and identically distributed with values in  $\{-1, 1\}$ . We are interested in the eigenvalues of the linear operator pencil

$$\mathcal{P}_c = \mathcal{P}_c(\lambda) = H_c - \lambda D_{\mp}.$$

Since the spectrum of the pencil  $\mathcal{P}_c(\lambda)$  equals that of the non-self-adjoint matrix  $D_{\mp}^{-1}H_c$ , one can see that we consider a tri-diagonal non-self-adjoint random matrix which has a fixed sign in each row. The case when  $c = 0$  was studied previously in [1,3], and similar problems can be found in [2,4]. Our purpose is to illustrate some numerical experiments in order to investigate the behaviour of the non-real eigenvalues of  $\mathcal{P}_c(\lambda)$  for different values of  $c$ .

### REFERENCES

- [1] S. N. Chandler-Wilde, R. Chonchaiya, and M. Lindner, *On the spectra and pseudospectra of a class of non-self-adjoint random matrices and operators*, arXiv preprint, arXiv:1107.0177, 2011.
- [2] E. B. Davies, *Spectral properties of random non-self-adjoint matrices and operators*, Proceedings of the Royal Society of London. Series A: Mathematical, Physical and Engineering Sciences, 457(2005), 191-206 (2001).
- [3] J. Feinberg, A. Zee, Physical Review E, 59(6), 6433, (1999).
- [4] I.Y. Goldsheid, B.A. Khoruzhenko, Israel Journal of Mathematics, 148(1), 331-346 (2005).

## Oscillation Criterion for Differential Equations with Several Delays

Nurten KILIÇ<sup>1</sup>

<sup>1</sup>*Department of Mathematics, Faculty of Science and Arts,  
Kütahya Dumlupınar University, Kütahya-TURKEY*

[nurten.kilic@dpu.edu.tr](mailto:nurten.kilic@dpu.edu.tr)

### ABSTRACT

In this paper, we concern with oscillatory behaviour of delay differential equations with several deviating arguments and obtain a new oscillation condition which improves some well known criteria in the literature. Also, we present an example to illustrate the result.

### REFERENCES

- [1] H. Akca, G.E. Chatzarakis, I.P. Stavroulakis, *Applied Mathematics Letters*, 59, 101-108 (2016).
- [2] E. Braverman, G.E. Chatzarakis, I.P. Stavroulakis, *Adv. Difference Equ.*, 2016 (87), 1-18 (2016).
- [3] G. E. Chatzarakis, H. Péics, *Applied Mathematics Letters*, 68, 20-26 (2017).
- [4] L.H. Erbe, B.G. Zhang, *Differ. Integral Equ.*, 1, 305-314 (1988).
- [5] L.H. Erbe, Q. Kong, B.G. Zhang, *Oscillation Theory for Functional Differential Equations*, Marcel Dekker, New York, 1995.
- [6] N. Fukagai, T. Kusano, *Ann. Mat. Pura Appl.*, 136, 95-117 (1984).
- [7] I. Györi and G. Ladas, *Oscillation Theory of Delay Differential Equations with Applications*, Clarendon Press, Oxford, 1991.
- [8] J. Jaroš and I. P. Stavroulakis, *Rocky Mountain J. Math.*, 29, 139-145 (1999).
- [9] N. Kılıç, *Oscillatory behaviour of linear delay differential equation with nonmonotone delays*, 9th International Conference on Applied Analysis and Mathematical Modelling.
- [10] M. Kon, Y. G. Sficas, I.P. Stavroulakis, *Proc. Amer. Math. Soc.*, 128, 2989-2997 (2000).
- [11] R.G. Koplatadze, T.A. Chanturiya, *Differentsial'nye Uravneniya*, 8, 1463-1465 (1982).
- [12] R. Koplatadze, G. Kvinikadze, *Georgian Mathematical Journal*, 1(6), 675-685 (1994).
- [13] M. K. Kwong, *J. Math. Anal. Appl.*, 156, 274-286 (1991).
- [14] G. Ladas and I. P. Stavroulakis. *J. Differential Equations*, 44, 134-152 (1982).
- [15] G. S. Ladde, V. Lakshmikantham, B. G. Zhang, *Oscillation Theory of Differential Equations with Deviating Arguments, Monographs and Textbooks in Pure and Applied Mathematics*, vol. 110, Marcel Dekker, Inc., New York, 1987.
- [16] J. S. Yu, Z. C. Wang, *Bull. Aust. Math. Soc.*, 46, 149-157 (1992).
- [17] J. S. Yu, Z. C. Wang, B.G. Zhang, X. Z. Qian, *PanAmerican Math. J.*, 2, 59-78 (1992).

## Instruction of Mathematics in Higher Education in the Covid-19 Pandemic

Mehmet Alper ARDIÇ<sup>1</sup>

<sup>1</sup>*Department of Mathematics, Faculty of Science and Arts,  
Adiyaman University, Adiyaman-TURKEY*

[mardic@adiyaman.edu.tr](mailto:mardic@adiyaman.edu.tr)

### ABSTRACT

The purpose of this study is to determine how mathematics instruction is carried out at the higher level of education in Covid-19 pandemic and what are the effects of these teaching activities. In the study, the case study model was used in which 30 lecturers with expertise in mathematics from 20 different universities in Turkey participated. In the study, the opinion form was used as a data collection tool. Descriptive analysis and content analysis methods were used in analyzing the data. It was found that before the pandemic, the use of technology in mathematics education by the lecturers was quite rare and at a basic level. In the pandemic process, it was observed that lecturers conducted their teaching synchronously or asynchronously with the traditional teaching approach, as before the pandemic, with distance education as the teacher-centered approach. It was understood that the main problems encountered during distance education were the difficulties encountered in teacher-student interaction and the inability to carry out assessment and evaluation activities in a healthy manner. It has been observed that the problems encountered especially in assessment and evaluation make it difficult to understand the actual impact of teaching mathematics during the pandemic on student learning. In fact, it was found that at the beginning of the pandemic, universities gave various instructions to their lecturers various trainings on distance education. However, these instructions were usually technical in scope and insufficient to overcome the difficulties encountered in the process, so that mathematics instruction was carried out with an understanding of 'emergency distance education' rather than formal distance education. On the other hand, the experience of distance education gained during the pandemic process had a positive influence on the views of a significant proportion of the lecturers on the integration of technology in mathematics education.

## Factorization of a Class of Multilinear Maps through Convolution Product

Ezgi ERDOĞAN<sup>1</sup>

<sup>1</sup>*Department of Mathematics, Faculty of Science and Arts,  
Marmara University, Istanbul-TURKEY*

[ezgi.erdogan@marmara.edu.tr](mailto:ezgi.erdogan@marmara.edu.tr)

### ABSTRACT

In this presentation, we give a factorization of a class of multilinear operators called zero product preserving maps through convolution product. We are interested with the multilinear operators defined on topological product of Banach algebras of integrable functions. This factorability allows us to investigate some properties of zero product preserving multilinear operators for certain domains such as compactness and summability. Finally, we present some isomorphisms between zero product preserving multilinear maps and n-homogeneous polynomials as an application of the factorization.

### REFERENCES

- [1] J. Alaminos, J. Extremera, A.R. Villena, Math. Proc. Cambridge Philos. Soc., 158(3), 493-504 (2015).
- [2] Y. Benyamini, S. Lassalle, J.G. Llavona, Bull Lond Math Soc. Bull. London Math. Soc., 38(3), 459-469 (2006).
- [3] K. Boulabiar, G. Buskes, Comm. Algebra, 34(4), 1435-1442 (2006).
- [4] G. Buskes, A. van Rooij, Positivity 4(3), 227-231 (2000).
- [5] G. Buskes, A. van Rooij, Rocky Mountain J. Math., 31(1), 45-56 (2001).
- [6] A. Ibort, P. Linares, J.G. Llavona, Rev. Mat. Complut. 25(1), 21-30 (2012).
- [7] E. Erdoğan, J.M. Calabuig E.A. Sanchez Perez, Ann Funct Anal., 9(2), 166-179 (2018).
- [8] E. Erdoğan, Ö. Gök, Indag. Math. (N.S.) 29(5), 1334-1349 (2018).
- [9] D. Perez-Garcia, I. Villanueva, J. Math. Anal. Appl. 306(1), 97-105 (2005).

## On the Floquet Solutions of the Second Order Linear Differential Equation with Almost Periodic Coefficients

Ashraf D. ORUJOV<sup>1</sup>

<sup>1</sup>Department of Mathematics and Science Education, Faculty of Education  
Sivas Cumhuriyet University, Sivas-TURKEY

[eorucov@hotmail.com](mailto:eorucov@hotmail.com)

### ABSTRACT

This study deals with the Floquet solutions of the differential equation

$$\ell_{\lambda}(y) = y'' + q_1(x)y' + [\lambda^2 + \lambda q_2(x) + q_3(x)]y = 0, x \in \mathbb{R}, \lambda \in \mathbb{C} \quad (1)$$

where the coefficients  $q_k(x) = \sum_{n=1}^{\infty} q_{kn} e^{i\alpha_n x}$ ,  $k = 1, 2, 3$  are almost periodic function and  $\{\alpha_n\}_{n=1}^{\infty}$  is an increasing sequence of positive numbers with  $\alpha_n \rightarrow +\infty$  and the set  $\{\alpha_n : n \in \mathbb{N}\}$  is an additive semigroup.

More general form of equation (1) and a special case  $\alpha_n = n$ ,  $n \in \mathbb{N}$  have been investigated in [1], [2] respectively.

Theorem: Let  $q_k(x)$ ,  $k = 1, 2, 3$  be uniform almost periodic functions and series  $\sum_{n=1}^{\infty} \alpha_n |q_{kn}|$ ,  $\sum_{n=1}^{\infty} |q_{3n}|$ ,  $k = 1, 2$  be convergent. Then for any  $\lambda \in \mathbb{C}, \lambda \neq \pm \alpha_n/2, n \in \mathbb{N}$  the differential equation  $\ell_{\lambda}(y) = 0, x \in \mathbb{R}$  has the Floquet solutions

$$f_1(x, \lambda) = e^{i\lambda x} \left( 1 + \sum_{n=1}^{\infty} U_n^{(1)} e^{i\alpha_n x} + \sum_{n=1}^{\infty} \frac{1}{\alpha_n + 2\lambda} \sum_{k=n}^{\infty} U_{nk}^{(1)} e^{i\alpha_k x} \right),$$
$$f_2(x, \lambda) = e^{-i\lambda x} \left( 1 + \sum_{n=1}^{\infty} U_n^{(2)} e^{i\alpha_n x} + \sum_{n=1}^{\infty} \frac{1}{\alpha_n - 2\lambda} \sum_{k=n}^{\infty} U_{nk}^{(2)} e^{i\alpha_k x} \right)$$

where the series  $\sum_{n=1}^{\infty} \alpha_n^2 |U_n^{(j)}|$  and  $\sum_{n=1}^{\infty} \frac{1}{\alpha_n} \sum_{k=n}^{\infty} \alpha_k^2 |U_{nk}^{(j)}|$ ,  $j = 1, 2$  converges.

Theorem follows that for any  $\lambda \in \mathbb{C} \setminus \{0\}, \lambda \neq \pm \alpha_n/2, n \in \mathbb{N}$  the functions  $f_1(x, \lambda), f_2(x, \lambda)$  are linearly independent solutions of the differential equation (1). Using these solutions, linearly independent solutions of the differential equation (1) corresponding to values  $\lambda = 0, \lambda = \pm \alpha_n/2, n \in \mathbb{N}$  can be established.

### REFERENCES

- [1] A. D. Orujov, Int. J. Pure Appl. Math., 26(2), 195-204 (2006).
- [2] Ashraf D. Orujov, Boundary Value Problems, 2015:117, 1-16 (2015).

## Numerical Solutions of 1D and 2D Sinh-Gordon Equation via Linear Barycentric Collocation Method

Ömer ORUÇ<sup>1</sup>

<sup>1</sup>*Department of Mathematics, Faculty of Science,  
Dicle University, Diyarbakır-TURKEY*

[omer.oruc@dicle.edu.tr](mailto:omer.oruc@dicle.edu.tr)

### ABSTRACT

We consider one-dimensional (1D) and two-dimensional (2D) Sinh-Gordon equation which is a nonlinear partial differential equation and has many applications in various branches of science. To obtain numerical solution of the Sinh-Gordon equation we use a centered finite difference method for temporal variable and employ linear barycentric collocation method for space variables. The obtained numerical results indicate that the proposed method in this work gives accurate results and it is applicable for such problems.

### REFERENCES

- [1] A.M. Wazwaz, Appl. Math. Lett., 25, 2354-2358 (2012).
- [2] M. Dehghan, M. Abbaszadeh, A. Mohebbi, Eng. Anal. Bound. Elem., 51, 220-235 (2015).
- [3] Ö. Oruç, Commun. Nonlinear. Sci. Numer. Simul., 57, 14-25 (2018).
- [4] J.-P. Berrut, R. Baltensperger, H.D. Mittelman, Internat. Ser. Numer. Math., 15, 27-51 (2005).
- [5] J.-P. Berrut, R. Baltensperger, BIT Numer. Math., 41, 868-879 (2001).



## Asymptotical Deferred Invariant Statistical Equivalence of Order $\alpha$ for Sequences of Sets

Esra GÜLLE<sup>1</sup> and Uğur ULUSU<sup>2</sup>

<sup>1</sup>*Afyon Kocatepe University, Afyonkarahisar-TURKEY*

<sup>2</sup>*Sivas Cumhuriyet University, Sivas-TURKEY*

[egulle@aku.edu.tr](mailto:egulle@aku.edu.tr)

[ugurulusu@cumhuriyet.edu.tr](mailto:ugurulusu@cumhuriyet.edu.tr)

### ABSTRACT

In this paper, we introduced the concepts of asymptotical strongly  $p$ -deferred invariant and deferred invariant statistical equivalence of order  $\alpha$  ( $0 < \alpha \leq 1$ ) in the Wijsman sense for sequences of sets. Also, we examined some relations between these concepts and investigated some properties of them.

### REFERENCES

- [1] R. P. Agnew, *Ann. of Math.*, 33, 413-421 (1932).
- [2] M. Altınok, B. İnan and M. Küçükcaslan, *Turk. J. Math. Comput. Sci.*, 3, 1-9 (2015).
- [3] G. Beer, *Set-Valued Anal.*, 2, 77-94 (1994).
- [4] M. Altınok, B. İnan and M. Küçükcaslan, *Thai J. Math.*, 18, 803-817 (2020).
- [5] F. Nuray, *J. Comput. Sci. & Comput. Math.*, 10, 1-6 (2020).

## Integral Inequalities for Differentiable $s$ –convex Functions in the Second Sense via Atangana-Baleanu Integral Operators

Merve AVCI ARDIÇ<sup>1</sup>, Ahmet Ocak AKDEMİR<sup>2</sup> and Havva KAVURMACI ÖNALAN<sup>3</sup>

<sup>1</sup>*Department of Mathematics, Faculty of Science and Arts,  
Adıyaman University, Adıyaman-TURKEY*

<sup>2</sup>*Department of Mathematics, Faculty of Science and Arts,  
Ağrı İbrahim Çeçen University, Ağrı-TURKEY*

<sup>3</sup>*Department of Mathematics Education, Faculty of Education  
Van Yüzüncü Yıl University, Van-TURKEY*

[mavci@adiyaman.edu.tr](mailto:mavci@adiyaman.edu.tr)  
[aocakakdemir@gmail.com](mailto:aocakakdemir@gmail.com)  
[havvaonalan@yyu.edu.tr](mailto:havvaonalan@yyu.edu.tr)

### ABSTRACT

Fractional integral operators, which form strong links between fractional analysis and integral inequalities, make unique contributions to the field of inequality theory due to their properties and strong kernel structures. In this context, the novelty brought to the field by the study can be expressed as the new and first findings of Ostrowski type that contain Atangana-Baleanu fractional integral operators for differentiable  $s$ -convex functions in the second sense. In the study, two new integral identities were established for Atangana-Baleanu fractional integral operators and by using these two new integral identities, Ostrowski type integral inequalities were obtained. In the findings, it was aimed to contribute to the field due to the structural properties of Atangana-Baleanu fractional integral operators.

### REFERENCES

- [1] Özdemir, M. E., Latif, M. A., & Akdemir, A. O. (2012). On some Hadamard-type inequalities for product of two  $s$ -convex functions on the co-ordinates. *Journal of Inequalities and Applications*, 2012(1), 1-13.
- [2] Özdemir, M. E., Kavurmaci, H., Akdemir, A. O., & Avci, M. (2012). Inequalities for convex and  $s$ -convex functions on  $\Delta=[a, b] \times [c, d]$ . *Journal of Inequalities and Applications*, 2012(1), 1-19.
- [3] Özdemir, M. E., Yildiz, C., & Akdemir, A. O. (2012). On some new Hadamard-type inequalities for co-ordinated quasi-convex functions. *Hacettepe Journal of Mathematics and Statistics*, 41(5), 697-707.
- [4] Butt, S. I., Akdemir, A. O., Nadeem, M., Mlaiki, N., İmdat, İ., & Abdeljawad, T. (2021).  $\$(m, n)$   $\$$ -Harmonically polynomial convex functions and some Hadamard type inequalities on the co-ordinates. *AIMS Mathematics*, 6(5), 4677-4690.
- [5] Set, E., Sarıkaya, M. Z., & Akdemir, A. O. (2014). Hadamard type inequalities for  $\varphi$ -convex functions on the co-ordinates. *Tbilisi Mathematical Journal*, 7(2), 51-60.
- [6] Ekinci, A., Akdemir, A. O., & Özdemir, M. E. (2017, April). On Hadamard-type inequalities for co-ordinated  $r$ -convex functions. In *AIP Conference Proceedings* (Vol. 1833, No. 1, p. 020118). AIP Publishing LLC.
- [7] J. Pečarić, F. Proschan, Y.L. Tong, *Convex Functions, Partial Orderings and Statistical Applications*, Academic Press, 1992.

## The Representation and Finite Sums of the Padovan- $p$ Jacobsthal Numbers

Özgür ERDAĞ<sup>1</sup> and Ömür DEVECİ<sup>1</sup>

<sup>1</sup>*Department of Mathematics, Faculty of Science and Letters,  
Kafkas University, Kars-TURKEY*

[ozgur\\_erdag@hotmail.com](mailto:ozgur_erdag@hotmail.com)

[odeveci36@hotmail.com](mailto:odeveci36@hotmail.com)

### ABSTRACT

In [1], Aküzüm defined the Padovan- $p$  Jacobsthal sequence  $\{J_n^p\}$  by the following homogeneous linear recurrence relation for any given  $p(p = 3, 4, \dots)$  and  $n \geq 0$

$$J_{n+p+4}^p = J_{n+p+3}^p + 3J_{n+p+2}^p - J_{n+p+1}^p - 2J_{n+p}^p + J_{n+2}^p - J_{n+1}^p - 2J_n^p$$

in which  $J_0^p = J_1^p = \dots = J_{p+2}^p = 0$  and  $J_{p+3}^p = 1$ . In this study, we derive the permanental and the determinantal representations of the Padovan- $p$  Jacobsthal numbers by using certain matrices which are obtained from the generating matrix of the Padovan- $p$  F Jacobsthal sequence. Furthermore, we obtain the combinatorial and exponential representations and the sums of the Padovan- $p$  Jacobsthal numbers by the aid of the generating function and the generating matrix of the Padovan- $p$  Jacobsthal sequence.

### REFERENCES

- [1] Y. Aküzüm, The Padovan- $p$  Jacobsthal Numbers and their Binet Formulas, is submitted.

## Directional Developable Surfaces in Euclidean 3-Space

Yanlin LI<sup>1</sup>, Rashad A. ABDEL-BAKY<sup>2</sup>

<sup>1</sup>*School of Mathematics, Hangzhou Normal University, Hangzhou-CHINA*

<sup>2</sup>*Department of Mathematics, Faculty of Science, University of Assiut,  
Assiut 71516, EGYPT*

[liy@hznu.edu.cn](mailto:liy@hznu.edu.cn)

### ABSTRACT

In this work we introduce a new version of developable ruled surfaces in the Euclidean 3-space. We establish an adapted frame along a spatial curve, and denote this the quasi-frame. We then introduce a parametric representation of a developable ruled surface, and call it a directional developable ruled surface. We investigate the uniqueness and the singularities of such developable surfaces.

### ACKNOWLEDGEMENT

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### REFERENCES

- [1] H. Pottmann, G. Farin, *Computer-Aided Geometric Design*, 12, 513-531 (1995).
- [2] H. Pottmann, M. Wallner, *Computer-Aided Geometric Design*, 16(6), 539-556 (1999).
- [3] H.Y. Zhao, G.J. Wang, *Progress in Nature Science*, 18, 105-110 (2008).
- [4] C.Y. Li, R.H. Wang, C.G. Zhu, *Applied Mathematics and Computation*, 218(7), 3199-3208 (2011).
- [5] C.Y. Li, R.H. Wang, C.G. Zhu, *Computer-Aided Design*, 43 1110-1117 (2011).
- [6] C.Y. Li, R.H. Wang, C.G. Zhu, *Computer-Aided Design*, 45, 621-627 (2013).

## Fuzzy Parameterized Intuitionistic Fuzzy Soft Topological Spaces

Adem YOLCU<sup>1</sup> and Taha Yasin ÖZTÜRK<sup>1</sup>

<sup>1</sup>*Department of Mathematics, Faculty of Science and Arts,  
Kafkas University, Kars-TURKEY*

[yolcu.adem@gmail.com](mailto:yolcu.adem@gmail.com)  
[taha36100@hotmail.com](mailto:taha36100@hotmail.com)

### ABSTRACT

Many scientists make an effort to simulate situations with unclear data. However, traditional approaches do not always succeed in expressing uncertainty. One of these approaches is the fuzzy parameterized intuitionistic fuzzy soft set (FPIFS) theory. This theory was introduced in 2019 by Sulukan et al. It has been defined and applied successfully on decision making problems. In this paper, we present fuzzy parameterized intuitionistic fuzzy soft topological spaces, as well as several associated notions and fundamental operations on this concept.

### REFERENCES

- [1] L. A. Zadeh, Information and Control, 8(3), 338–353 (1965).
- [2] K. T. Atanassov, Fuzzy Sets and Systems, 20(1), 87-96 (1986).
- [3] D.A. Molodtsov, Computers and Mathematics with Applications, 37(4-5), 19-31 (1999).
- [4] E. Sulukan, N. Cagman, T. Aydın, Journal of New Theory, 29, (2019), 79-88.

## Parameter Dependent Refinements of Hölder and Minkowski's Inequalities

Khuram Ali KHAN<sup>1</sup> and Asima IQBAL<sup>1</sup>

<sup>1</sup>*Department of Mathematics, University of Sargodha, Sargodha 40100, PAKISTAN*

[khuramsms@gmail.com](mailto:khuramsms@gmail.com)

[iaa351527@gmail.com](mailto:iaa351527@gmail.com)

### ABSTRACT

In the field of mathematical analysis, Jensen's inequality plays a prominent role because of its wide range of applications. The discrete Jensen's inequality is a classical tool to study many other inequalities e.g, arithmetic mean, geometric mean inequality, Hölder and Minkowski inequalities. In this work, a new parameter-dependent refinement of the discrete Jensen's inequality is to be utilized. Some new quasi-arithmetic and mixed symmetric means are also studied with their monotonicity and convergence. Beck's inequalities are established for the given parameter dependent sequence to refine Hölder and Minkowski inequalities by using the convexity of function of several variables.

## On Important Fundamental Integral Inequalities For Convex Functions

Mehmet Zeki SARIKAYA<sup>1</sup> and Hüseyin BUDAK<sup>1</sup>

<sup>1</sup>*Department of Mathematics, Faculty of Science and Arts,  
Düzce University, Düzce-TURKEY*

[sarikayamz@gmail.com](mailto:sarikayamz@gmail.com)

[hsyn.budak@gmail.com](mailto:hsyn.budak@gmail.com)

### ABSTRACT

The main purpose of this paper is to acquaint the readers with some of the most significant theorems, that are widely used in the theory of inequalities. These are inequalities regarding so-called convex mappings. So at the end of this paper, readers should be able to know a general background of inequalities and overview of relation between convex functions and inequalities.

### REFERENCES

- [1] M. Alomari, M. Darus, U.S. Kirmaci, *Computers and Math. with Appl.*, 59, 225-232 (2010).
- [2] S.S. Dragomir, R.P. Agarwal, *Appl. Math. Lett.*, 11(5), 91-95 (1998).
- [3] D.A. Ion, *Annals of University of Craiova Math. Comp. Sci. Ser.*, 34, 82-87 (2007).
- [4] J. L. W. V. Jensen, *Nyt. Tidsskr. Math. B.*, 16, 49-69 (1905).
- [5] C.E.M. Pearce, J. Pecaric, *Appl. Math. Lett.*, 13(2), 51-55 (2000).
- [6] U.S. Kirmaci, *Appl. Math. Comp.*, 147, 137-146 (2004).
- [7] U.S. Kirmaci, M.E. Özdemir, *Appl. Math. Comp.*, 153, 361-368 (2004).
- [8] J. Pecaric, F. Proschan, Y.L. Tong, *Convex functions, partial ordering and statistical applications*, Academic Press, New York, 1991.
- [9] D.S. Mitrinovic, J. E. Pecaric and A. M. Fink, *Inequalities involving functions and their integrals and derivatives*, Kluwer Academic Publishers, Dordrecht, 1991.
- [10] A.M. Ostrowski, *Comment. Math. Helv.*, 10, 226-227 (1938).

## Eigenvalue Estimates over Compact Spacelike Spin Hypersurfaces of Lorentzian Manifolds

Serhan EKER<sup>1</sup>

<sup>1</sup>*Department of Mathematics, Faculty of Science and Arts,  
Ağrı İbrahim Çeçen University, Ağrı-TURKEY*

[serhane@agri.edu.tr](mailto:serhane@agri.edu.tr)

### ABSTRACT

In this work, Yongfa Chen's estimates for the eigenvalues of the Dirac-Witten operator of compact (with or without boundary) spacelike hypersurfaces of Lorentzian manifold is improved in terms of the scalar curvature, mean curvature, Energy-Momentum tensor and its trace. Then we give some equalities according to the first eigenvalue of the Dirac operator.

### ACKNOWLEDGEMENT

This study was supported by TUBITAK The Scientific and Technological Research Council of Turkey (Project Number: 120F109).

### REFERENCES

- [1] E.C. Kim, Bull.Korean Math. Soc. 46 (5), 949-966 (2009).
- [2] T. Friedrich, *Grauate Studies in Mathematics, American Mathematical Society*, 25 (2000).
- [3] T.Friedrich, and E.C. Kim, Journal of Geometry and Physics, 37(1-2), 1-14 (2000).
- [4] O. Hijazi. Commun. Math. Phys., 104, 151-162 (1986).
- [5] X. Zhang, Mathematical Research Letters, 5(2), 199-210 (1998).
- [6] C. YongFa, Science in China Series A: Mathematics 52(11), 2459-2468 (2009).



## Some Estimates on the Spin Manifolds by Using $\beta$ –twist Dirac operator

Serhan EKER<sup>1</sup>

<sup>12</sup>*Department of Mathematics, Faculty of Science and Arts,  
Ağrı İbrahim Çeçen University, Ağrı-TURKEY*

[serhane@agri.edu.tr](mailto:serhane@agri.edu.tr)

### ABSTRACT

In this work we obtain optimal estimates in terms of the trace of nondegenerate Codazzi tensor with the help of the  $\beta$  –twist Dirac operator.

### ACKNOWLEDGEMENT

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### REFERENCES

- [1] C. Bar, Math. Ann. 239, 39-46 (1992).
- [2] E.C. Kim, Bull.Korean Math. Soc. 46 (5), 949-966 (2009).
- [3] T. Friedrich, *Grauate Studies in Mathematics, American Mathematical Society*, 25 (2000).
- [4] T.Friedrich, and E.C. Kim, Journal of Geometry and Physics, 37(1-2), 1-14 (2000).
- [5] T. Friedrich, and E.C.Kim, Bull. Korean Math. Soc. 45(2), 365-373 (2008).
- [6] O. Hijazi. Commun. Math. Phys., 104, 151-162 (1986).
- [7] X. Zhang, Mathematical Research Letters, 5(2), 199-210 (1998).

## Laplace Type Integral Transforms for Solving Generalized Katugampola Fractional Kinetic Equations Involving Certain Class of Functions

Durmuş ALBAYRAK<sup>1</sup>

<sup>1</sup>*Department of Basic Sciences, Turkish Naval Academy,  
National Defence University, İstanbul-TURKEY*

[dalbayrak@dho.edu.tr](mailto:dalbayrak@dho.edu.tr)

### ABSTRACT

In this paper, we use  $\mathcal{L}_{\alpha,\mu}$ -transform which is a generalization of Laplace Transform to solve fractional kinetic energy equations involving certain class of functions. Fractional integrals that appear in these equations are defined in the sense of Katugampola fractional integrals. Firstly, We prove our main results regarding the solutions of some fractional kinetic energy equations, and then give examples to illustrate these results.

### REFERENCES

- [1] S.S. Dragomir and C.E.M. Pearce, *Selected Topics on Hermite-Hadamard Inequalities and Applications*, RGMIA Monographs, Victoria University, 2000.
- [2] S.S. Dragomir, *Comput. Math. Appl.* 38, 33-37 (1999).
- [3] D.S. Mitrinovic, J.E. Pecaric and A.M. Fink, *Classical and New Inequalities in Analysis*, Kluwer Academic Publishers, Dordrecht, 1993.
- [4] I. Podlubni, *Fractional Differential Equations*, Academic Press, San Diego, 1999.
- [5] M.Z. Sarıkaya, E. Set, H. Yaldız, N. Basak, *Mathematical and Computer Modelling* 57, 2403-2407 (2013).

## On Some Properties of Invariant Statistical Convergence in Fuzzy Normed Spaces

Şeyma YALVAÇ<sup>1</sup> and Erdiñç DÜNDAR<sup>1</sup>

<sup>1</sup>*Afyon Kocatepe University, Afyonkarahisar-TURKEY*

[syalvac@aku.edu.tr](mailto:syalvac@aku.edu.tr)

[edundar@aku.edu.tr](mailto:edundar@aku.edu.tr)

### ABSTRACT

In this study, we defined the concept of invariant statistical convergence in fuzzy normed spaces. Also, we investigated some properties such as uniqueness and linearity of this new concept in fuzzy normed spaces.

### REFERENCES

- [1] C. Felbin, Fuzzy Sets and Systems, 48, 293–248 (1992).
- [2] J.A. Fridy, Analysis, 5, 301-313 (1985).
- [3] M. Mursaleen, O. H. H. Edely, Appl. Math. Lett. 22, 1700–1704 (2009).
- [4] R.A. Raimi, Duke Math. J. 30, 81–94 (1963).
- [5] C. Şençimen and S. Pehlivan, Fuzzy Sets and Systems, 159, 361–370 (2008).
- [6] Ş. Yalvaç and E. Dündar, Honam Mathematical J. 43(3), 433–440 (2021).

## Concircular Curvature Tensor of Nearly Cosymplectic Manifolds with Generalized Tanaka-Webster Connection

Çağatay MADAN<sup>1</sup>, Gülhan AYAR<sup>1</sup> and Nesip AKTAN<sup>2</sup>

<sup>1</sup>*Department of Mathematics, Karamanoğlu Mehmetbey University,  
Karaman-TURKEY*

<sup>2</sup>*Department of Mathematics, Necmettin Erbakan University,  
Konya-TURKEY*

[cagataymadan@gmail.com](mailto:cagataymadan@gmail.com)

[gulhanayar@gmail.com](mailto:gulhanayar@gmail.com)

[nesipaktan@gmail.com](mailto:nesipaktan@gmail.com)

### ABSTRACT

The aim of this study is to research concircular curvature tensor of nearly cosymplectic manifolds with generalized Tanaka-Webster connection. With this study, we have focused on the important curvature properties of nearly cosymplectic manifolds equipped with Tanaka-Webster connection. Also, based on these curvature properties, we have defined the concircular curvature tensor with respect to the generalized Tanaka-Webster connection. Then, we emphasized the properties that concircular curvature tensor of nearly cosymplectic manifolds with Tanaka-Webster connection provides in case of flatness,  $\xi$ -concircularly flatness,  $\phi$ -concircularly semisymmetric.

### REFERENCES

- [1] N. Tanaka, Japanese Journal of Mathematics. New series, 2(1), 131-190 (1976).
- [2] S. Tanno Tohoku Mathematical Journal, Second Series, 21(1), 21-38 (1969).
- [3] S. Tanno, Transactions of the American Mathematical Society, 314(1), 349-379 (1989).
- [4] G Ghosh, D.U. Chand, Publications de l'Institut Mathematique, 102(116), 221-230 (2017).
- [5] H. Endo, Ann. St. Univ. Iasi, 51, 439-554 (2005).
- [6] D. Nicola, A. Dielo, G. Yudin, Annali di Matematica Pura ed Applicata 197(1),127-138 (2018).
- [7] D.E. Blair, *Riemannian Geometry of Contact and Symplectic Manifolds*, Progress in Math. 203, Birkhauser Boston 2002.

## Elastoplastic Complication for Perforated Plate in Transverse Shear

Rustam Mammadov<sup>1</sup> and Rafail Mekhtiyev<sup>2</sup>

<sup>1</sup>*Department of Composite materials,  
Azerbaijan Technical University, AZERBAIJAN*

<sup>2</sup>*Department of Applied Mathematics,  
Azerbaijan State Oil and Industry University, AZERBAIJAN*

[mrhammadov888@gmail.com](mailto:mrhammadov888@gmail.com)

### ABSTRACT

A solution is given to the problem of transverse shear of a thin plate clamped along the edges of the holes and weakened by a doubly periodic system of rectilinear through cracks with plastic end zones collinear to the abscissa and ordinate axes of unequal length. General representations of solutions are constructed that describe the class of problems with a doubly periodic stress distribution outside circular holes and rectilinear cracks with end zones of plastic deformations. Satisfying the boundary conditions, the solution of the problem of the theory of shear plates is reduced to two infinite systems of algebraic equations and two singular integral equations. Then each singular integral equation is reduced to a finite system of linear algebraic equations.

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## **Extensions of Discrete Majorization Type Inequalities for Convex Functions Defined on rectangles**

**Hidayat ULLAH<sup>1</sup> and M. Adil KHAN<sup>1</sup>**

*<sup>1</sup>Department of Mathematics, University of Peshawar, PAKISTAN*

[Hidayatmohmand4@gmail.com](mailto:Hidayatmohmand4@gmail.com)

[adilswati@gmail.com](mailto:adilswati@gmail.com)

### **ABSTRACT**

We establish several discrete majorization type inequalities for convex functions defined on rectangles. We obtain the intended inequalities while using the support line inequality, Chebyshev's inequality and the concept of co-ordinate convexity.

## Generalization of Jensen-Mercer Inequality and Hermite-Hadamard Inequality via Majorization

Shah FAISAL<sup>1</sup> and Muhammad Adil KHAN<sup>2</sup>

<sup>1,2</sup>*Department of Mathematics, University of Peshawar, PAKISTAN*

[sfaisaluop@gmail.com](mailto:sfaisaluop@gmail.com)

[madilkhan@uop.edu.pk](mailto:madilkhan@uop.edu.pk)

### ABSTRACT

The theory of majorization has been getting considerable attention of the researchers working in different fields. It has been used as key tool for solving complicated problems of optimization. The main theme of this article is to present generalized form of Jensen-Mercer inequality and Hermite-Hadamard inequality by using the concept of majorization. We establish generalized Mercer's inequality by considering majorized tuples and non-negative weights. By imposing strict condition of monotonicity on the tuples and relax condition on the weights, we obtain another result for Jensen-Mercer inequality. Furthermore, we also obtain generalized Hermite-Hadamard inequality for certian majorized tuples by applying the above obtained results.

## On Soliton Surface Associated with Betchow-Da Rios Equation in Minkowski Space

Yanlin LI<sup>1</sup>, Melek ERDOĞDU<sup>2</sup> and Ayşe YAVUZ<sup>3</sup>

<sup>1</sup>*Department of Mathematics, Hangzhou Normal University, Hangzhou, CHINA*

<sup>2</sup>*Department of Mathematics-Computer Sciences,  
Necmettin Erbakan University, Konya, TURKEY*

<sup>3</sup>*Department of Mathematics and Science Education,  
Necmettin Erbakan University, Konya, TURKEY*

[yanlinmath@163.com](mailto:yanlinmath@163.com)

[merdogdu@erbakan.edu.tr](mailto:merdogdu@erbakan.edu.tr)

[ayasar@erbakan.edu.tr](mailto:ayasar@erbakan.edu.tr)

### ABSTRACT

The main scope of this presentation is to investigate the soliton surfaces associated with the Betchow-Da Rios Equation in Minkowski space. We discuss the differential geometric properties of these kind of soliton surfaces with respect to the Lorentzian casual characterizations. Moreover, the linear maps of Weingarten type, defined on tangent space of these soliton surfaces, are stated. Finally, some new results are obtained by means of two geometric invariants  $k$  and  $h$  which are generated by linear maps of Weingarten type.

### REFERENCES

- [1] G. Ganchev, M. Velichka, Turkish Journal of Mathematics, 38(5),883-895 (2014).
- [2] M. Barros, A. Ferrández, P. Lucas, M. A. Meroño. Journal of Geometry and Physics, 31(2-3), 217-228 (1999).
- [3] M. Barros, A. Ferrández, P. Lucas, M. A. Meroño, *Solutions of the Betchov- Da Rios soliton equation in the anti-De Sitter 3-space*, New Approaches in Nonlinear Analysis, Hadronic Press, Palm Harbor, FL, 51-71,1999.
- [4] C. Rogers, W.K. Schief, *Backlund and Darboux Transformations: Geometry of Modern Applications in Soliton Theory*, Cambridge University Press, 2002.



## Decision-Making Approach with Bipolar Fuzzy Soft Graphs

Yıldırım ÇELİK<sup>1</sup>

<sup>1</sup>*Department of Mathematics, Faculty of Science and Arts,  
Ordu University, Ordu-TURKEY*

[ycelik61@gmail.com](mailto:ycelik61@gmail.com)

### ABSTRACT

In this study, we give a novel frame work for bipolar fuzzy soft graphs and present some basic notions on it. We also develop efficient algorithm to solve multiple criteria decision-making problem regarding detection of bipolar disorder in children by using bipolar fuzzy soft graphs.

### REFERENCES

- [1] S. Abdullah M. Aslam, K. Ullah, Journal of Intelligent and Fuzzy System, 27(2), 729-742 (2014).
- [2] M. Akram, Information Sciences, 177, 5548-5564 (2011).
- [3] Y. Çelik, Creative Mathematics and Informatics, 27, 123-132 (2018).
- [4] P.K. Maji, A.R. Roy and R. Biswas, Journal of Fuzzy Mathematics, 9(3), 589-602 (2001).
- [5] D.A. Molodtsov, Computers and Mathematics with Applications, 37, 19-31 (1999).
- [6] J.N. Mordeson and C.S. Peng, Information Sciences, 79, 159-170 (1994).
- [7] S. Shahzadi and M. Akram, Journal of Applied Mathematics and Computing, 55(1-2), 369-392 (2017).
- [8] H.L. Yang, S.G. Li, L. Guo and C. H. Ma, Knowledge-Based Systems, 27, 60-68 (2012).
- [9] L.A. Zadeh, Information and Control, 8(3), 338-353 (1965).
- [10] L.A. Zadeh, Information Sciences, 3(2), 177-200 (1971).

## On Rough $\mathcal{J}$ -Convergence and Rough $\mathcal{J}$ -Cauchy Sequence in 2-Normed Spaces

Mukaddes ARSLAN<sup>1</sup> and Erdinç DÜNDAR<sup>1</sup>

<sup>1</sup>*Afyon Kocatepe University, Afyonkarahisar-TURKEY*

[mukad.deu@gmail.com](mailto:mukad.deu@gmail.com)

[edundar@aku.edu.tr](mailto:edundar@aku.edu.tr)

### ABSTRACT

In this paper, firstly we introduced the concept of rough  $\mathcal{J}^*$ -convergence and investigated the relations between rough  $\mathcal{J}$ -convergence and rough  $\mathcal{J}^*$ -convergence in 2-normed spaces. Secondly, we defined the concept of rough  $\mathcal{J}$ -Cauchy sequence and examined the relations between rough  $\mathcal{J}$ -convergence and rough  $\mathcal{J}$ -Cauchy sequence in 2-normed spaces. Also, we introduced the concept of rough  $\mathcal{J}^*$ -Cauchy sequence and investigated the relations between rough  $\mathcal{J}$ -Cauchy sequence and rough  $\mathcal{J}^*$ -Cauchy sequence in 2-normed spaces.

### REFERENCES

- [1] M. Arslan, E. Dündar, Konuralp J. Math., 6(1), 57–62 (2018).
- [2] M. Arslan, E. Dündar, Bull. Math. Anal. Appl., 10(3), 1–9 (2018).
- [3] E. Dündar, C. Çakan, Gulf J. Math., 2(1), 45–51 (2014).
- [4] A. Nabiev, S. Pehlivan, M. Gürdal, Taiwanese J. Math., 11(2), 569–576 (2007).
- [5] H. X. Phu, Numer. Funct. Anal. Optim., 22, 199–222 (2001).

## H-symmetry in Kahler Manifolds

İnan ÜNAL<sup>1</sup>

<sup>1</sup>*Department of Computer Engineering, Faculty of Engineering  
Munzur University, Tunceli-TURKEY*

[inanunal@munzur.edu.tr](mailto:inanunal@munzur.edu.tr)

### ABSTRACT

H-curvature tensors have been defined on Kahler manifolds by using curvature properties of such manifolds. In complex geometry, Riemannian geometry of complex manifolds has been studied by using H-curvature tensors. In this study, we focus on Kahler manifolds under some semi-symmetry conditions related to H-curvature tensors.

### REFERENCES

- [1] B.B. Sinha, Tensor N.S. 23, 271–274 (1972).
- [2] H.C. Lal, S. Srivastava, Int. Assoc. Sci. Innov. Res. (IASIR) (IJTCAS 14-783), 10(3) (2014).
- [3] I. Hinterleitner, Filomat 30(11), 3115–3122 (2016).
- [4] A.T. Vanli, I. Unal, International Journal of Geometric Methods in Modern Physics, 15(12), 185-205 (2018).
- [5] S. Kobayashi, K. Nomizu, *Foundations of Differential Geometry* (Vol. 1, No. 2). New York, London, 1963.

## The Solution of Linear Volterra Integral Equation of The First Kind Aboodh Transform

Özgür KOTAN<sup>1</sup> and Ercan ÇELİK<sup>1</sup>

<sup>1</sup>*Department of Mathematics, Faculty of Science,  
Ataturk University, Erzurum-TURKEY*

[ozgur2525@gmail.com](mailto:ozgur2525@gmail.com)  
[ercelik@atauni.edu.tr](mailto:ercelik@atauni.edu.tr)

### ABSTRACT

In this paper, we apply Aboodh transform to solve linear Volterra integral equation of the first kind. A few examples solved by Aboodh Transform. Aboodh transform is a powerful method for solving linear Volterra integral equations of the first kind. The Convolution theorem for the Aboodh transform has been proved. Aboodh transform for the solution of linear Volterra integral equation of the first kind presented and in application section of this paper.

### REFERENCES

- [1] S. Aggarwal, A.R. Gupta D.P. Singh, International Journal of Latest Technology in Engineering, Management & Applied Science, 7(9), 141-145 (2018).
- [2] K.S. Aboodh, R.A. Farah, I.A. Almarady, Solving delay differential equations by Aboodh transformation method. International Journal of Applied Mathematics &
- [3] S. Aggarwal, R. Chauhan, N. Sharma, International Journal of Latest Technology in Engineering, Management & Applied Science, 7(4), 138-140 (2018).
- [4] M. Mohand, M.A. Mahgob, T.M. Elzaki, Global Journal of Pure and Applied Mathematics, 11(3), 1179-1184 (2015).
- [5] R. Najafi, G.D. Küçük, E. Çelik, Mathematical Methods in the Applied Sciences, 40(4), 939-946 (2017).

## On Optimal Control of the Initial Velocity of an Euler-Bernoulli Beam System

Arif ENGİN<sup>1</sup>, Yeşim SARAÇ<sup>1</sup> and Ercan ÇELİK<sup>1</sup>

<sup>1</sup>Department of Mathematics, Faculty of Science,  
Atatürk University, Erzurum-TURKEY

[arif.engin@ogr.atauni.edu.tr](mailto:arif.engin@ogr.atauni.edu.tr)

[ysarac@atauni.edu.tr](mailto:ysarac@atauni.edu.tr)

[ercelik@atauni.edu.tr](mailto:ercelik@atauni.edu.tr)

### ABSTRACT

We examine an optimal control problem governed by the Euler-Bernoulli beam equation. The initial velocity of the system is given by the control function. We give sufficient conditions for the existence of a unique solution of the hyperbolic system and prove that the optimal solution for the considered optimal control problem exists and is unique. We derive the gradient of the cost functional to be minimized via an adjoint problem. Finally, we furnish some numerical examples to demonstrate the effectiveness of the results obtained.

### REFERENCES

- [1] A. Hasanov, H.A. Ituo, *Applied Mathematics Letters*, 87, 141-146 (2019).
- [2] A. Hasanov, A. Kawano, *Inverse Problems*, 32, 1-31 (2016).
- [3] A. Kowalewski, *Optimal Control Via Initial Conditions of a Time Delay Hyperbolic System*, The 18<sup>th</sup> International Conferences on Methods and Models in Automation and Robotics, 22-25 August, Miedzyzdroje, Poland, 2011.
- [4] B.Z. Guo, W. Kang, *International Journal of Control*, 87, 925-939 (2013).
- [5] B. Kundu, R. Ganguli, *Mathematics and Computation*, 298, 247-260 (2017).

## **Evolving Evolutoids and Pedaloids of Curves from Viewpoints of Envelope and Singularity Theory in Minkowski Plane**

**Zhichao YANG<sup>1</sup> and Yanlin LI<sup>1</sup>**

<sup>1</sup>*School of Mathematics, Hangzhou Normal University, Hangzhou-CHINA*

[liy1@hznu.edu.cn](mailto:liy1@hznu.edu.cn)

### **ABSTRACT**

The notions of evolutoids and pedaloids are generalized by evolutes and pedals. There are already some results about the relationships between these curves in the Euclidean plane and Minkowski plane. However, because lightlike vectors exist, the Minkowski plane situation is quite different from the Euclidean plane. Therefore, researchers defined two kinds of evolutoids and pedaloids to obtain the relationships like in the Euclidean plane. In this paper, we give a new method to overcome the obstacle and do not need to define two types of evolutoids and pedaloids. We initially use envelope theory to study the evolutoids and pedaloids in the Minkowski plane, illustrating an internal correlation between algebraic and geometric viewpoints, and give the geometric explanation of evolutoids and pedaloids. Then we generalize the notions of evolutoids and pedaloids to the category of frontal in the Minkowski plane. Furthermore, we apply the technique of singularity theory, using the discriminants and versal unfolding tools, to consider evolving evolutoids and give singularity types of the evolutoids, and explain when cusps and inflexions occur and how evolutoids evolve. Besides, there is a close relationship between the evolutoids and pedaloids and a good correspondence between their singularities. Finally, we give an example to show our results.

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### **REFERENCES**

- [1] V.I. Arnold, Proc. Steklov Inst. Math., 226, 20-28 (1999).
- [2] G.A. Sekerci, S. Izumiya, Bull. Malays. Math. Sci. Soc., (2021) .
- [3] S. Izumiya, N. Takeuchi, Note di Matematica, 39, 13-24 (2020).

## A Note On Beams and Inextensible Flows in Trans-Sasakian Manifolds

Azime ÇETİNKAYA<sup>1</sup>

<sup>1</sup>*Maritime Higher Vocational School, Piri Reis University, Istanbul-TURKEY*

[acetinkaya@pirireis.edu.tr](mailto:acetinkaya@pirireis.edu.tr)

### ABSTRACT

We find the classification of beams and inextensible flows of curves in Trans-Sasakian manifolds. We mention some results for special type of curves. Also we investigate about inextensible flows of curves in submanifolds of Trans-Sasakian manifolds. We give examples.

### REFERENCES

- [1] D.E. Blair, *Contact manifolds in Riemannian geometry*, Lecture Notes in Mathematics, Vol. Springer-Verlag, Berlin-New York, 1976.
- [2] D.Y. Kwon, F.C. Park, *Appl. Math. Lett.*, 12, 115-119 (1999).
- [3] J.C. Marrero, *Ann. Mat. Pura Appl.*, (4) 162, 77-86 (1992).
- [4] J.A. Oubina, *Publ. Mat. Debrecen*, 32(3-4), 187-193 (1985).
- [5] K. Yano, M. Kon, *Structure on manifolds*, World Scientific, 1984.

## The Expansion Formula for a class of Dirac Operators with a Spectral Parameter in Boundary Condition

Aynur ÇÖL<sup>1</sup> and Khanlar R. MAMEDOV<sup>2</sup>

<sup>1</sup>*Department of Mathematics and Science Education,  
Sinop University, Sinop-TURKEY*

<sup>2</sup>*Department of Mathematics, Faculty of Science and Arts,  
Iğdır University, Iğdır-TURKEY*

[aynurcol@sinop.edu.tr](mailto:aynurcol@sinop.edu.tr)  
[hanlar.residoglu@igdir.edu.tr](mailto:hanlar.residoglu@igdir.edu.tr)

### ABSTRACT

We consider the boundary value problem generated by the Dirac equations system

$$By' + mTy + \Omega(x)y = \lambda y, \quad 0 \leq x < \infty \quad (1)$$

and boundary condition

$$(\alpha_0 + \alpha_1 \lambda + \alpha_2 \lambda^2)y_1(0) - (\beta_0 + \beta_1 \lambda + \beta_2 \lambda^2)y_2(0) = 0, \quad (2)$$

where  $m > 0$  is a mass and  $\lambda$  is a spectral parameter,

$$y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, B = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, T = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \Omega(x) = \begin{pmatrix} p(x) & q(x) \\ q(x) & -p(x) \end{pmatrix},$$

$p(x)$  and  $q(x)$  are real measurable functions such that

$$|p(x)| \leq \frac{c}{(1+x)^{2+\varepsilon}}, \quad |q(x)| \leq \frac{c}{(1+x)^{1+\varepsilon}},$$

here  $c$  and  $\varepsilon$  are positive numbers. The numbers  $\alpha_i, \beta_i \in \mathbb{R}$  ( $i = 0, 1, 2$ ) satisfy the conditions  $\alpha_1\beta_0 > \alpha_0\beta_1, \alpha_2\beta_1 > \alpha_1\beta_2, \alpha_2\beta_0 = \alpha_0\beta_{12}$ .

In this work, the operator interpretation of the boundary value problem (1)-(2) is investigated. The resolvent operator is constructed and the expansion formula with respect to eigenfunctions is obtained.

### REFERENCES

- [1] M. G. Gasymov, Trudy Moskov. Mat. Obšč, 19, 41–112 (1968).
- [2] A. Çöl, Kh. R. Mamedov, J. Math. Anal. Appl. 393, 470–478 (2012).
- [3] Kh. R Mamedov, A. Çöl, International Journal of Computational Cognition, 7(4), 20-24 (2009).



## Investment and Oil Price Affect on Gross Domestic Product of Azerbaijan

Elnure SHAFIZADE<sup>1</sup>

<sup>1</sup>*Department of Economics and Business Management, UNEC Business School,  
Azerbaijan State University of Economics (UNEC), Bakü-AZERBAIJAN*

[elnure\\_sh@unec.edu.az](mailto:elnure_sh@unec.edu.az)

### ABSTRACT

In this work, was created the dynamic model that demonstrates the dependence of GDP on investments and oil price in case of Azerbaijan economy.

This approach provides an opportunity for strategic planning of GDP for the country. In this work, to achieve the desired level of GDP, the volume of investment and oil price are used as the independent variable in the dynamic model. But as indicated above, many other factors affect GDP. We chose two of them: the amount of investment and oil price. But even so, the dynamic model of the optimal GDP trajectory yielded good results.

Further research will take into account the other most influential factors on GDP. In this case, a dynamic model of the optimal trajectory of GDP will give even more adequate results.

### REFERENCES

- [1] F.A. Aliyev, *Metody resheniya prikladnykh zadach optimizachii dinamicheskikh system*, Baku, (1989). (Aliiev F.A., Solution methods of optimization dynamic systems applied problems, Baku, 1989) (in Russian)
- [2] A. Brayson, Kho Yu-Shi, *Prikladnaya teoriya optimalnogo upravleniya*, Moskva, (1972). (Bryson A., Ho Yu-Chi, Applied Optimal Control, Moscow, 1972) (in Russian).
- [3] E.R. Shafizade, G. Hasanova, *Dynamic model for Gross Domestic Product in Azerbaijan*, 6th International Conference on Control and Optimization with Industrial Applications, COIA-2018, 11-13 July, Baku, Azerbaijan (SCA) (2018).
- [4] E.R. Shafizade, *Econometric model for gross domestic product in Azerbaijan*, 55th International Scientific Conference on Economic and Social Development was dedicated to Azerbaijan State University of Economics 90th anniversary, Book of Proceedings Vol. 3/4, Baku, pp.160-166, (2020).
- [5] E.R. Shafizade, N.R. Aslanova, *Forecasting model for gross domestic product in Azerbaijan*, 55th International Scientific Conference on Economic and Social Development was dedicated to Azerbaijan State University of Economics 90th anniversary, Book of Proceedings Vol. 2/4, Baku, pp.134-142, (2020).
- [6] E.R. Shafizade, *The dynamic model of the optimal trajectory to achieve the desired level of GDP for Azerbaijan*, Proceedings of the 7th International Conference on Control and Optimization with Industrial Applications, Volume II. Baku, Azerbaijan, pp.353-355, (2020)
- [7] E.R. Shafizade, A.B. Ramazanov, *Diskretnaya dinamicheskaya model VVP Azerbaydjana*, Proceedings of IAM, V.7. N.2., pp.270-284, (2018). (Shafizade, E.R., Ramazanov, A.B. (2018). *Discrete dynamic model of Azerbaijan's GDP*. Proceedings of IAM. V.7. N.2. pp.270-284.).

## The Expansion Formula for a class of Dirac Operators with Discontinuous Coefficient and Spectral Parameter in Boundary Condition

Aynur ÇÖL<sup>1</sup> and Khanlar R. MAMEDOV<sup>2</sup>

<sup>1</sup>*Department of Mathematics and Science Education,  
Sinop University, Sinop-TURKEY*

<sup>2</sup>*Department of Mathematics, Faculty of Science and Arts,  
İğdir University, İğdir-TURKEY*

[aynurcol@sinop.edu.tr](mailto:aynurcol@sinop.edu.tr)  
[hanlar.residoglu@igdir.edu.tr](mailto:hanlar.residoglu@igdir.edu.tr)

### ABSTRACT

On the semi-infinite interval  $[0, \infty)$ , we consider the Dirac differential equation system

$$By' + \Omega(x)y = \lambda\rho(x)y, \quad 0 \leq x < \infty \quad (1)$$

with the boundary condition

$$p_1(\lambda)y_1(0) - p_2(\lambda)y_2(0) = 0, \quad (2)$$

where

$$y = \begin{pmatrix} y_1(x) \\ y_2(x) \end{pmatrix}, B = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \Omega(x) = \begin{pmatrix} p(x) & q(x) \\ q(x) & -p(x) \end{pmatrix},$$

$p(x)$  and  $q(x)$  are real measurable functions such that the condition

$$\int_0^\infty \|\Omega(x)\| dx < \infty \quad (3)$$

is satisfied for the Euclidean norm of the matrix function  $\Omega(x)$ , the coefficient  $\rho(x)$  is a piecewise constant function which takes the form

$$\rho(x) = \begin{cases} \alpha, & 0 \leq x < a, \\ 1, & a \leq x < \infty \end{cases}$$

and  $1 \neq \alpha > 0$ . Here,  $\lambda$  is a spectral parameter and  $p_i(\lambda)$  ( $i = 1, 2$ ) is a quadratic polynomial

$$p_1(\lambda) = \alpha_0 + \alpha_1 \lambda + \alpha_2 \lambda^2, \quad p_2(\lambda) = \beta_0 + \beta_1 \lambda + \beta_2 \lambda^2$$

with

$$\alpha_1 \beta_0 > \alpha_0 \beta_1, \alpha_2 \beta_1 > \alpha_1 \beta_2, \alpha_2 \beta_0 = \alpha_0 \beta_{12}, \alpha_i, \beta_i \in \mathbb{R} \quad (i = 0, 1, 2).$$

In this paper, the operator interpretation of the boundary value problem (1)-(3) is examined. The resolvent operator is obtained and the expansion formula is given with respect to eigenfunctions for the considered problem.

### REFERENCES

- [1] I.M. Guseĭnov, Doklady Akademii Nauk Azerbaĭdzhana, 55(1-2), 13-18 (1999).
- [2] Kh. R. Mamedov, A. Çöl, Math. Meth. Appl. Sci., 35, 1712–1720 (2012).
- [3] Kh. R. Mamedov, A. Çöl, International Journal of Computational Cognition, 7(4), 20-24 (2009).

The Expansion Formula for a class of Dirac Operators with Discontinuous Coefficient and Spectral Parameter in Boundary Condition

Aynur ÇÖL and Khanlar R. MAMEDOV – Oral Presentation / 065

## On the Parabola as a Quadratic Bezier Curve in $E^2$

Şeyda KILIÇOĞLU<sup>1</sup>, and Süleyman ŞENYURT<sup>2</sup>

<sup>1</sup>*Department of Mathematics, Faculty of Education  
Baskent University, Ankara-TURKEY*

<sup>2</sup>*Department of Mathematics, Faculty of Science and Arts,  
Ordu University, Ordu-TURKEY*

[seyda@baskent.edu.tr](mailto:seyda@baskent.edu.tr)  
[senyurtsuleyman52@gmail.com](mailto:senyurtsuleyman52@gmail.com)

### ABSTRACT

It is well known that a quadratic Bézier curve is also a parabolic segment of a main parabola. In this study first we have examined the matrix representation of a parabola segment as a special planar quadratic Bezier curve in  $E^2$ . Also we have given some properties and found the vertex of parabola  $y=ax^2+bx+c$  which has the parabolic segment as a quadratic Bézier curve with given control points.

### REFERENCES

- [1] A. Levent, B. Sahin, Proceedings of the Institute of Mathematics and Mechanics, National Academy of Sciences of Azerbaijan, 44(2), 222-228 (2008).
- [2] "Derivatives of a Bézier Curve" <https://pages.mtu.edu/~shene/COURSES/cs3621/NOTES/spline/Bezier/bezier-der.html>.
- [3] D. Marsh, Applied Geometry for Computer Graphics and CAD. Springer Science and Business Media., 2006.
- [4] F. Tas, K. Ilarslan, International Journal of Geometric Methods in Modern Physics, 16(6), (2019).
- [5] G. Farin, *Curves and Surfaces for Computer-Aided Geometric Design*, Academic Press, 1996.
- [6] H. Zhang, F. Jieqing, *Bezier Curves and Surfaces (2)*, State Key Lab of CAD & CG Zhejiang University, 2006.
- [7] H. Hagen, Rocky Mountain J. Math., 16(3), 629-638 (1986).
- [8] S. Michael, Bezier curves and surfaces, Lecture 8, Floater Oslo Oct., 2003.
- [9] Ş. Kılıçoğlu, S. Şenyurt, Ordu University Journal of Science and Technology, 9(2), 83-97 (2019).
- [10] Ş. Kılıçoğlu, S. Şenyurt, European Journal of Pure and Applied Mathematics, 13, 216-226 (2020).
- [11] Ş. Kılıçoğlu, S. Şenyurt, On the Matrix Representation of 5th order Bézier Curve and derivatives. Communications Faculty of Sciences University of Ankara Series A1 Mathematics and Statistic. (accepted) 2021.

## An examination on to find 5th order Bézier curve in $E^3$

Şeyda KILIÇOĞLU<sup>1</sup> and Süleyman ŞENYURT<sup>2</sup>

<sup>1</sup>*Department of Mathematics, Faculty of Education  
Baskent University, Ankara-TURKEY*

<sup>2</sup>*Department of Mathematics, Faculty of Science and Arts,  
Ordu University, Ordu-TURKEY*

[seyda@baskent.edu.tr](mailto:seyda@baskent.edu.tr)  
[senyurtsuleyman52@gmail.com](mailto:senyurtsuleyman52@gmail.com)

### ABSTRACT

In this study we have examined the way to find the 5th order Bezier curve based on the control points with matrix form, while the first, the second and the third derivatives are given in  $E^3$ . Also we give an example to find the 5th order Bezier curve with given derivatives.

### REFERENCES

- [1] A. Levent, B. Sahin, Proceedings of the Institute of Mathematics and Mechanics, National Academy of Sciences of Azerbaijan, 44(2), 222-228 (2008).
- [2] "Derivatives of a Bézier Curve" <https://pages.mtu.edu/~shene/COURSES/cs3621/NOTES/spline/Bezier/bezier-der.html>.
- [3] D. Marsh, Applied Geometry for Computer Graphics and CAD. Springer Science and Business Media., 2006.
- [4] F. Tas, K. Ilarslan, International Journal of Geometric Methods in Modern Physics, 16(6), (2019).
- [5] G. Farin, *Curves and Surfaces for Computer-Aided Geometric Design*, Academic Press, 1996.
- [6] H. Zhang, F. Jieqing, *Bezier Curves and Surfaces (2)*, State Key Lab of CAD & CG Zhejiang University, 2006.
- [7] H. Hagen, Rocky Mountain J. Math., 16(3), 629-638 (1986).
- [8] S. Michael, Bezier curves and surfaces, Lecture 8, Floater Oslo Oct., 2003.
- [9] Ş. Kılıçoğlu, S. Şenyurt, Ordu University Journal of Science and Technology, 9(2), 83-97 (2019).
- [10] Ş. Kılıçoğlu, S. Şenyurt, European Journal of Pure and Applied Mathematics, 13, 216-226 (2020).
- [11] Ş. Kılıçoğlu, S. Şenyurt, On the Matrix Representation of 5th order Bézier Curve and derivatives. Communications Faculty of Sciences University of Ankara Series A1 Mathematics and Statistic. (accepted) 2021.

## Flat Translation Surfaces with Respect to Semi-Symmetric Connections in $E^3$ and $E_1^3$

Mustafa ALTIN<sup>1</sup>

<sup>1</sup>*Technical Sciences Vocational School, Bingöl University, Bingöl-TURKEY*

[maltin@bingol.edu.tr](mailto:maltin@bingol.edu.tr)

### ABSTRACT

In this paper, we calculate Gaussian curvature of translation surfaces in  $E^3$  and  $E_1^3$  endowed with a certain semisymmetric (non-)metric connection. After, we classify flat translation surfaces with respect to semi-symmetric metric connections and semi-symmetric non-metric connections in  $E^3$  and  $E_1^3$ .

### REFERENCES

- [1] A. Levent, B. Sahin, Proceedings of the Institute of Mathematics and Mechanics, National Academy of Sciences of Azerbaijan, 44(2), 222-228 (2008).
- [2] N.S. Agashe, M.R. Chafle, Indian J. Pure Appl. Math., 23, 399-409 (1992).
- [3] N.S. Agashe, M.R. Chafle, Tensor, 55, 120-130 (1994).
- [4] M. E. Aydin and A. Mihai, Proc. Rom. Acad. Ser. A, Math. Phys. Tech. Sci. Inf. Sci., 16(4), 477-483 (2015).
- [5] H.A. Hayden, Proc. London Math. Soc., 34, 27-50 (1932).
- [6] T. Imai, Tensor, 23, 300-306 (1972).
- [7] T. Imai, Tensor, 24, 293-296 (1972).
- [8] I. Inoguchi, R. Lopez and M.I. Munteanu, Geom. Dedicata, 16, 221-231 (2012).
- [9] H. Liu, Y. Yu, Proc. Japan Acad. Ser. A, 89(9), 111-113 (2013).
- [10] R. Lopez, Beitrage Alge. Geom., 52, 105-112 (2011).
- [11] R. Lopez, M.I. Munteanu, J. Math. Soc. Jpn., 64(3), 985-1003 (2012).
- [12] M. Moruz, M.I. Munteanu, J Math. Analysis Appl., 439, 798-812 (2016).
- [13] Z. Nakao, Proc. Amer. Math. Soc., 54, 261-266 (1976).
- [14] K. Seo, Osaka J. Math., 50, 631-641 (2013).
- [15] D. Yang and Y. Fu, J. Math. Analysis Appl., 440, 437-450 (2016).
- [16] K. Yano, Rev. Roumaine Math. Pures Appl., 15, 1579-1586 (1970).
- [17] D. W. Yoon, Taiwanese J. Math., 17(5), 1545-1556 (2013).
- [18] D.W. Yoon, Open Math., 15(1), 459-466 (2017).
- [19] D.W Yoon, C. W. Lee, M.K. Karacan, Bull. Korean Math. Soc., 50(4), 1329-1343 (2013).

## The Differential Examination with Blaschke Approach Between The Spacelike Curves and The Spacelike Ruled Surfaces

Muradiye ÇİMDİKER ASLAN<sup>1</sup> and Yasin ÜNLÜTÜRK<sup>1</sup>

<sup>1</sup>Department of Mathematics, Faculty of Science and Arts,  
Kirkklareli University, 39100, Kirkklareli-TURKEY

[muradiyecimdiker@klu.edu.tr](mailto:muradiyecimdiker@klu.edu.tr)  
[yasinunluturk@klu.edu.tr](mailto:yasinunluturk@klu.edu.tr)

### ABSTRACT

In this work, by means of the Frenet and Blaschke frame, we study a system of differential equations which is establish between the spacelike curves and the spacelike ruled surface in dual Lorentz space and obtain the solutions of these systems for special cases. Furthermore, Regarding to these special solutions, we give Darboux rotation vector, the distribution parameters, the pitch of the spacelike ruled surfaces which is generated by the dual spacelike curve.

### REFERENCES

- [1] N. Ayyıldız, A.C. Çöken, A. Yücesan, Balk. J. Geo. Appl., 7(1), 1-12 (2001).
- [2] N. Ayyıldız, A.C. Çöken, A. Kılıç, Tensor N.S., 62(2), 112-119 (2000).
- [3] W. Blaschke, *Vorlesungen über differential geometrie I*, Verlag Von Julius Springer-Verlag in Berlin, 1930.
- [4] C. Ekici, Ü.Z. Savcı, Y. Ünlütürk, Math. Sci and Appl. E-Notes, 1(1) 79-89 (2013).
- [5] H.W. Guggenheimer, *Differential geometry*, Mc. Graw-Hill Book Company, New York, 1963.
- [6] S. Nizamoğlu, N. Gülpınar, J. Fac. Scie Ege Uni., 16(1), 53-62 (1993).
- [7] B. O'Neill, *Semi-Riemannian geometry with applications to relativity*, Academic Press Inc. London, 1983.
- [8] Ü. Pekmen, J. Fac. Scie. Ege Uni., 16(1), 67-74 (1995).
- [9] E. Study, *Die geometrie der dynamen*, Verlag Teubner, Leipzig, 1933.
- [10] A. Tutar, O. Şener, IJST A3 (Special issue-Mathematics), 327-330 (2012).
- [11] H.H. Uğurlu, A. Çalışkan, *Darboux ani dönme vektörleri ile spacelike ve timelike yüzeyler geometrisi*, CBÜ Yay. Manisa, 2012.
- [12] G.R. Veldkamp, Mech. Math. Theory, 11, 141-156 (1976).

## Curves of Constant Breadth with B-Darboux Frame in Euclidean 3-Space

Muradiye ÇİMDİKER ASLAN<sup>1</sup>

<sup>1</sup>*Department of Mathematics, Faculty of Science and Arts,  
Kirkklareli University, 39100, Kirkklareli-TURKEY*

[muradiyecimdiker@klu.edu.tr](mailto:muradiyecimdiker@klu.edu.tr)

### ABSTRACT

In this paper, firstly, curves of constant breadth with B-Darboux frame are introduced in Euclidean 3-space. Then some geometric properties the curves of constant breadth with B-Darboux frame are studied.

### REFERENCES

- [1] B. Altunkaya, F.K. Aksoyak, Commun. Fac. Sci. Uni. Ank. Series A1, 66(2), 44-52 (2017).
- [2] N.H. Ball, American Math. Monthly, 37(3), 348-353 (1930).
- [3] M. Dede, M. Çimdiker Aslan, C. Ekici, Math. Methods in Appl. Sci., (2021).
- [4] G. Darboux, *Leçons sur la Theorie Generale des Surfaces I-II-III-IV*, Gauthier-Villars, Paris, 1896.
- [5] M. Fujiwara, Tohoku Math. J., 5, 179-184 (1914).
- [6] Ö. Köse, Doğa Math., 8, 119-126 (1984).
- [7] Ö. Köse, Doğa Math., 10, 11-14 (1986).
- [8] A. Mağden, Ö. Köse, Turk. J. Math., 21, 277-284 (1997).
- [9] B. O'Neill, *Semi-Riemannian geometry with applications to relativity*, Academic Press Inc. London, 1983.
- [10] G. Uğur Kaymanlı, J. of New Theory, 33, 50-55 (2020).
- [11] B. Uzunoğlu, İ. Gök, Y. Yaylı, App. Math. Comput., 275, 317-323 (2016).

## Nearly Kenmotsu Manifolds Endowed with a Semi-Symmetric Metric Connection

Mustafa YILDIRIM<sup>1</sup>

<sup>1</sup>*Department of Mathematics, Faculty of Science and Arts,  
Aksaray University, Aksaray-TURKEY*

[mustafayldrm24@gmail.com](mailto:mustafayldrm24@gmail.com)

### ABSTRACT

This paper deals with the classification of a nearly Kenmotsu manifold satisfying certain geometric conditions. We obtain sufficient conditions for a nearly Kenmotsu manifold of dimension  $2n+1$  endowed with a semi-symmetric metric connection.

### REFERENCES

- [1] D. E. Blair, *Riemannian Geometry of Contact and Symplectic Manifolds*, vol. 203 of *Progress in Mathematics*, Birkhäuser, Boston, Mass, USA, 2nd edition, 2010.
- [2] D.E. Blair, D.K. Showers, K. Yano, *Kodai Math.*, 175-180 (1976).
- [3] T.Q. Binh, *Periodica Mathematica Hungarica* 21, 101–107 (1990).
- [4] K. Kenmotsu, *Tohoko Math. J.*, 24, 93-103 (1972).
- [5] A. Shukla, *Kuwait J. Sci. Eng.*, 23(2), 139-144 (1996).



## A Study on the Applications of the Generalized Exponential Rational Function Method

Serbay DURAN<sup>1</sup> and Birgül BİNZET<sup>2</sup>

<sup>1</sup>*Department of Mathematics and Science Education, Faculty of Education,  
Adiyaman University, Adiyaman TURKEY*

<sup>2</sup>*Department of Mathematics, Faculty of Science and Arts,  
Adiyaman University, Adiyaman-TURKEY*

[sduran@adiyaman.edu.tr](mailto:sduran@adiyaman.edu.tr)

[bpeker@adiyaman.edu.tr](mailto:bpeker@adiyaman.edu.tr)

### ABSTRACT

Modelling of physical phenomena with many applications in fluid dynamics is described with the help of nonlinear differential equations. Many analytical methods have been developed to obtain exact solutions to these equations. Among these analytical methods, the analysis of traveling wave solutions obtained by using the generalized exponential rational function method will be included. In addition, the advantages and disadvantages of the applied method compared to other methods are discussed.

### REFERENCES

- [1] B. Ghanbari, M.S. Osman, D. Baleanu, *Modern Physics Letters A*, 34(20), 1950155 (2019).
- [2] S. Duran, *Optical and Quantum Electronics*, 53(8), 1-9 (2021).
- [3] H. Durur, A. Yokuş, *Mathematical Sciences*, 1-11 (2021).
- [4] S. Duran, A. Yokuş, H. Durur, D. Kaya, *Modern Physics Letters B*, 35(26), 2150363 (2021).

## Fixed Point Theorems in b-Metric Spaces with Applications

Monica BOTA<sup>1,3</sup>, Liliana GURAN<sup>2</sup> and Adrian PETRUȘEL<sup>1</sup>

<sup>1</sup>*Department of Mathematics, Faculty of Mathematics and Computer Science,  
Babeș-Bolyai University, Cluj-Napoca-ROMANIA*

<sup>2</sup>*Department of Farmaceutical Sciences, Western Vasile Goldiș University of Arad*

<sup>3</sup>*Academy of Romanian Scientists, 3 Ilfov str, Bucharest, ROMANIA*

[monica.bota@ubbcluj.ro](mailto:monica.bota@ubbcluj.ro)  
[lguran@uvvg.ro](mailto:lguran@uvvg.ro)  
[adrian.petrusel@ubbcluj.ro](mailto:adrian.petrusel@ubbcluj.ro)

### ABSTRACT

We establish some fixed point theorems for Ciric- operators in the context of b-metric spaces. The starting point of our research was a Ran-Reurings theorem, which gave a recent research direction in fixed point theory. The Ulam-Hyers stability of the fixed point problem, data dependence and well-posedness are also discussed. The results are applied to a coupled fixed point problem.

### REFERENCES

- [1] Lj. Ciric, Publ. Inst. Math. Beograd, 12(26), 19-26 (1971) .
- [2] S. Czerwik, Acta Mathematica et Informatica Univ. Ostraviensis, 1, 5-11 (1993).
- [3] D. Guo, V. Lakshmikantham, Nonlinear Anal., 11, 623-632 (1987).
- [4] R. Miculescu, A. Mihail, J. Fixed Point Theory Appl. 19, 2153-2163 (2017).
- [5] A. Petrușel, G. Petrușel, B. Samet, J.-C. Yao, Fixed Point Theory, 17, 459-478 (2016).

## Information Security as an Important Factor in the Use of Digital Technologies

Sevda HAJIZADE<sup>1</sup> and Sevda BADALOVA<sup>1</sup>

<sup>1</sup>*Department of digital technologies and applied informatics,  
Azerbaijan State University of Economics, Baku-AZERBAIJAN*

[s.hajizada@unec.edu.az](mailto:s.hajizada@unec.edu.az)  
[sevdabadalova17@gmail.com](mailto:sevdabadalova17@gmail.com)

### ABSTRACT

The rates and proportions of dynamic and innovative development have come to the fore among the factors that support the competitiveness of economic entities. Digital technology plays a decisive role in this regard. The intensification of digital transformations in the context of a pandemic, along with positive aspects, has also raised some problems. Among them, information security plays a particularly important role.

The article points out the need to ensure the necessary security for the exchange of information and knowledge in the development of the digital economy. The systemic measures developed and implemented in the field of ICT in Azerbaijan were commented on. The environment for the use of digital technologies in the country was described as one of the important factors that increase the competitiveness of the national economy. The place and role of information protection in building the information society have been determined. In order to strengthen the export-oriented progressive ICT potential, the requirements for the training of highly qualified specialists in the field of information security have been clarified. The process of building an electronic state in Azerbaijan was characterized, the issues of information security of it and the business environment were studied. In the context of the expansion of the scope of online activities and a sharp increase in the number of virtual offices, the necessity of taking preventive measures from the point of view of information security was substantiated, proposals were made on the organizational and economic aspects of their implementation.

## Double-Diffusive Free Convection Flow of a Rate Type Fluid

Azhar Ali ZAFAR<sup>1</sup> and Jan AWREJCEWICZ<sup>2</sup>

<sup>1</sup> *Department of Mathematics, Government College University Lahore-PAKISTAN*

<sup>2</sup> *Lodz University of Technology Lodz, POLAND*

[azharalizafar@gcu.edu.pk](mailto:azharalizafar@gcu.edu.pk)

[jan.awrejcewicz@p.lodz.pl](mailto:jan.awrejcewicz@p.lodz.pl)

### ABSTRACT

This paper presents the time dependent behaviour of double-diffusive free convection flow of an electrically conducting, incompressible Maxwell fluid over a moving vertical plate in the presence of external magnetic field that is fixed or moves along with the plate. Heat transfer analysis is carried out by taking thermal conductivity as an exponential function of time, constant concentration and first order chemical reaction. The non-dimensionalized partial differential equations are solved by the Laplace transform method. An interesting property regarding the behaviour of the fluid velocity is found when the magnetic field moves with the plate. In this case the fluid velocity is not zero far away of the plate. Mechanical, thermal and concentration effects on the fluid motion are separately brought to light. Moreover, the details of flow and heat transfer characteristics and their dependence on some of the physical parameters are drawn out by graphical illustrations. Furthermore, particular cases of the motion of the plate are also discussed.

### ACKNOWLEDGEMENT

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### REFERENCES

- [1] N. Ghara, S. Das, S. L. Maji, R. N. Jana, *Am. J. Sci. Ind. Res.*3, 376–386 (2012).
- [2] R. Nandkeolyar, M. Das, P. Sibanda, *Boundary Value Problems*, 247, 1-21 (2013).
- [3] M. Narahari, L. Debnath, *Angew. Math. Mech.*, 93, 38-49 (2013).
- [4] J. N. Tokis, *Astrophys. Space Sci.*, 112, 413-422 (1985).

## Fixed Point Result On Complete Metric Space with Respect to W-Distance

Liliana GURAN<sup>1</sup> and Abdelkader BELHENNICHE<sup>2,3</sup>

<sup>1</sup> *Departement of Pharmaceutical Sciences, Faculty of Pharmacy, Vasile Goldis Western University. Arad, L. Rebreanu Street, no. 86, 310048,*

<sup>2</sup> *SYSTEC – Research Center for Systems and Technologies, Faculty of Electrical Engineering, Porto University, Institute of Systems and Technologies, 4200-465, Porto, PORTUGAL*

<sup>3</sup> *Laboratoire des Études pratiques en sciences de gestion et sciences commerciales. École supérieure de commerce, 42003 Kolea, Tipaza, ALGERIE*

[guran.lilian@uvvg.ro](mailto:guran.lilian@uvvg.ro)  
[belhenniche@fe.up.pt](mailto:belhenniche@fe.up.pt)

### ABSTRACT

In this article we prove the existence of fixed point theorem in a complete metric space with respect to w-distance by resorting to variational methods such us: Ekeland's variational principle theorem and Directional contraction of Clarcke.

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### REFERENCES

- [1] F. H. Clarke, Nonsmooth analysis and optimization. Proceedings of the international congress of mathematicians. Vol. 5. 1983.
- [2] O. Kada, T. Suzuki and W. Takahashi, Math. Japonica, 44, 381-391(1996).
- [3] L. Guran, Carpathian Journal of Mathematics, 89-92 (2007).
- [4] M.S. Kumar, Bull. Math. Anal. Appl., 3(2), 134-139 (2011).
- [5] I. Ekeland, Journal of Mathematical Analysis and Applications, 47, 324-353 (1974).

## Mathematical Model of Crack Formation and Growth in Composite Material During Single-Axis Tension

Rafael MEHTIYEV<sup>1</sup>, Nurlan GURBANOV<sup>2</sup> and Kamala ISMAYILOVA<sup>2</sup>

<sup>1</sup>*Department of General and Applied Mathematics, Faculty of Information Technology and Control, Azerbaijan State Oil and Industry University, Baku-AZERBAIJAN*

<sup>2</sup>*Department of Materials Science and Processing Technologies, Faculty of Oil Mechanical Engineering, Baku-AZERBAIJAN*

[rafail.mehtiyev.k@asoiu.edu.az](mailto:rafail.mehtiyev.k@asoiu.edu.az)

[nurlan.gurbanov@asoiu.edu.az](mailto:nurlan.gurbanov@asoiu.edu.az)

[kamala.ismayilova@asoiu.edu.az](mailto:kamala.ismayilova@asoiu.edu.az)

### ABSTRACT

This paper proposes a mathematical model that describes the formation and growth of nanochats in nanocrystalline material. In the model, the concentration of stresses at the apex of the resulting elliptical crack causes dislocations at the end joints of the grain boundaries, causing the grain boundaries to shift. The stress field created by these dislocations and the load applied to the crack ends cause nanochats to form and grow. Mathematical calculations show that an increase in the radius of curvature of elliptical cracks and a decrease in grain size contribute to the growth of nanochats. These cases are consistent with experimental data on low shear resistance and plasticity values for most hybrid nanocrystalline materials.

### CONCLUSION

Theoretical analysis shows that the formation of nanochats can lead to the growth of wounds in the grains and grain boundaries in deformed nanocrystalline metals. Model used Border sliding during loading leads to the formation of nanochats. It turns out that the nucleation and growth of nanocranes in nanocrystalline materials is more associated with an increase in the radius of curvature  $\rho$  of the crack tip.

### REFERENCES

- [1] J.D. Kuntz, G.-D. Zhan, A.K. Mukherjee MRS Bull. 29 (2004) 22. Зернограничное проскальзывание и зарождение нанотрещин... 45
- [2] D. Wolf, V. Yamakov, S.R. Phillpot, A.K. Mukherjee, H. Gleiter, Acta Mater. 53 (2005).
- [3] M. Dao, L. Lu, R.J. Asaro, J.T.M. De Hosson, E. Ma, Acta Mater. 55 (2007).
- [4] C.C. Koch, I.A. Ovid'ko, S. Seal, S. Veprek, Structural Nanocrystalline Materials: Fundamentals and Applications (Cambridge University Press, Cambridge, 2007).
- [5] A. Mukhopadhyay, B. Basu. Int. Mater. Rev. 52 (2007)
- [6] I.A. Ovid'ko. Int. Mater. Rev. 50, (2005)
- [7] M.Yu. Gutkin, I.A. Ovid'ko. Phys. Solid State 52 (2010).

## Applications of Uhlmann Phase of Quantum Mixed State in Topological Phase Transition

Xu-Yang HOU<sup>1</sup>, Hao GUO<sup>1</sup> and Chih-Chun CHIEN<sup>2</sup>

<sup>1</sup> Department of Physics, Southeast University, Jiulonghu Campus, Nanjing 211189, CHINA

<sup>2</sup> Department of Physics, University of California, Merced, Merced, California 95343, USA

[230189154@seu.edu.cn](mailto:230189154@seu.edu.cn)

### ABSTRACT

The Uhlmann process is built on the density matrix of a mixed quantum state and offers a way to characterize topological properties at finite temperatures<sup>1-3</sup>. In the process of researching the Loschmidt-amplitude, we find that the Loschmidt-amplitude zero of the quantum mixed state can be used to characterize the topological phase transition in the Uhlmann process. We present examples of Cruetz-ladder model and three-level systems exhibiting finite-temperature TQPTs associated with the Loschmidt-amplitude zeros<sup>4</sup>. Especially, We analyze an ideal spin- $j$  quantum paramagnet in a magnetic field undergoing an Uhlmann process and derive general formulas of the Uhlmann phase and Loschmidt amplitude for arbitrary  $j$  as the system traverses a great circle in the parameter space<sup>5</sup>. The exact results of  $j = 0.5$  and  $j = 1$  systems show topological regimes that survive only at finite temperatures but not at zero temperature, and the number of TQPTs is associated with the winding number in the parameter space. Finally, we discuss the possible experimental schemes of spin- $j$  system undergoing Uhlmann process. Our results pave the way for future studies on finite-temperature topological properties, and possible experimental protocols.

### REFERENCES

- [1] A. Uhlmann, Rep. Math. Phys. 24, 229 (1986).
- [2] A. Uhlmann, Ann. Phys. (Berlin) 501, 63 (1989).
- [3] A. Uhlmann, Lett. Math. Phys. 21, 229 (1991).
- [4] X.-Y. Hou, Q.-C. Gao, H. Guo, Y. He, T. Liu, and C. C. Chien, Phys. Rev. B 102, 104305 (2020).
- [5] X.-Y. Hou, H. Guo, and C. C. Chien, Phys. Rev. A 104, 023303 (2021).

## Late-Order Asymptotics of Ordinary Differential Equations

Fatih SAY<sup>1</sup>

<sup>1</sup>*Department of Mathematics, Faculty of Arts and Sciences,  
Ordu University, Ordu-TURKEY*

[fatihsay@odu.edu.tr](mailto:fatihsay@odu.edu.tr)

### ABSTRACT

This talk presents the late-terms of the asymptotic series of the singular ordinary differential equation by its pre-factors [1]. It addresses some formal asymptotic results of asymptotic analysis [1,2,3,4].

### REFERENCES

- [1] F. Say, Hacettepe Journal of Mathematics and Statistics, 50(2), 342-350 (2021).
- [2] R. B. Dingle, *Asymptotic expansions: their derivation and interpretation*, Academic Press, London-New York, 1973.
- [3] M. V. Berry, *Asymptotics, superasymptotics, hyperasymptotics...* In *Asymptotics beyond all orders* (pp. 1-14). Springer, Boston, MA, 1991.
- [4] F. W. J. Olver, *Methods Appl. Anal.* 1(1), 1-13 (1994).



## Differential Game for the Pontryagin's Example with *Life-line*

Bahrom SAMATOV<sup>1</sup> and Ulmasjon SOYIBBOEV<sup>1</sup>

<sup>1</sup>*Department of Mathematical analysis, Faculty of Mathematics,  
Namangan State University, Namangan-UZBEKISTAN*

[samatov57@inbox.ru](mailto:samatov57@inbox.ru)  
[ulmasjonsoyibboev@gmail.com](mailto:ulmasjonsoyibboev@gmail.com)

### ABSTRACT

In this abstract, we have studied “Life-line” differential game of two players for the Pontryagin's control example. In this case, players move by controlled acceleration vectors. We have subjected Geometric constraints to controls of both players. In a pursuit problem,  $\Pi$  – strategy is proposed for Pursuer and by this strategy, pursuit conditions and guaranteed pursuit time are obtained. In an evasion problem, a special strategy is proposed for Evader and evasion conditions are generated. Using the  $\Pi$  – strategy, we have defined an attainability domain of Pursuer and shown that it is monotone decreasing with respect to inclusion in  $t$ . Moreover, an attainability set of Evader is constructed and “Life-line” problem is solved to the advantage of the Evader.

Consider the differential game when Pursuer  $X$  and Evader  $Y$  having radius vectors  $x$  and  $y$  correspondingly move in the space  $R^n$ . If their acceleration vectors are  $u$  and  $v$ , then the game will be described by the equations:  $\ddot{x} - a\dot{x} = u$ ,  $x(0) = x_0$ ,  $\dot{x}(0) = x_1$  (1) and  $\ddot{y} - a\dot{y} = v$ ,  $y(0) = y_0$ ,  $\dot{y}(0) = y_1$  (2), where  $x, y, u, v \in R^n$ ,  $n \geq 2$ ,  $a \neq 0$ ;  $x(0) = x_0$  and  $y(0) = y_0$  are the initial positions of the objects  $X$  and  $Y$ .  $\dot{x}(0) = x_1$  and  $\dot{y}(0) = y_1$  are the initial velocity vectors of the objects  $X$  and  $Y$ . It is assumed that  $x_0 \neq y_0$  and  $x_1 = y_1$ . Here the control function  $u$  satisfies a geometric constraint  $|u(t)| \leq \alpha$ , for almost every  $t \geq 0$  (3). The control function  $v$  satisfies a geometric constraint  $|v(t)| \leq \beta$ , for almost every  $t \geq 0$  (4). In the “Life-line” game (1)-(4), a closed set  $M \subset R^n$  is given and it is supposed that  $y_0 \notin M$ . The objective for Pursuer  $X$  is to capture Evader  $Y$ , i.e. to reach the equality  $x(t) = y(t)$  at some finite time  $t > 0$  while Evader  $Y$  is in the zone  $R^n/M$ . The aim of Evader  $Y$  is to reach the zone  $M$  before being caught by Pursuer  $X$  or to keep the relation  $x(t) \neq y(t)$  for all  $t \geq 0$ , and if it is impossible, to delay the moment of meeting as far as possible.

### REFERENCES

- [1] A.A. Azamov and B.T. Samatov, The  $\Pi$  – strategy: Analogies and Applications. The Fourth International Conference Game Theory and Management, St. Petersburg, Vol.4, P. 33-47, 2010.
- [2] R. Isaacs, *Differential games*, John Wiley and Sons, New York, 1965.
- [3] L.A. Petrosjan, *Differential games of pursuit. Series on optimization*, Vol.2. World Scientific Publishing, Singapore, 1993.

## **Automation of the Information-Measurement Process and Improving the Accuracy of Measurements**

**Almaz MEHDIYEVA<sup>1</sup>**

*<sup>1</sup>Department of Electronics and Automation,  
Azerbaijan State Oil and Industry University, Baku, AZERBAIJAN*

[almazmehdiyeva@yahoo.com](mailto:almazmehdiyeva@yahoo.com)

### **ABSTRACT**

At most enterprises of the technological process, automation occurs spontaneously, projects are financed from various sources, and development is carried out by unrelated development teams. And this leads to insufficient compatibility of operating systems, communications, applications, and storage formats and data management. Existing methods have been investigated in order to increase the accuracy of the measured quantities. These methods are justified in reducing either systematic or random error. Studies have shown that the finite difference filter reduces both errors. In order to reduce the measurement error, it is proposed to use a finite difference filter.

## Jensen–Mercer and Related Inequalities on Fractal Sets

Saad Ihsan BUTT<sup>1</sup> and Saba YOUSAF<sup>1</sup>

<sup>1</sup>*Department of Mathematics, COMSATS University Islamabad, Lahore Campus, PAKISTAN*

[saadihsanbutt@gmail.com](mailto:saadihsanbutt@gmail.com)

[sabayousafsh9@gmail.com](mailto:sabayousafsh9@gmail.com)

### ABSTRACT

The most notable inequality pertaining convex functions is Jensen's inequality which has tremendous applications in several fields. Mercer introduced an important variant of Jensen's inequality called as Jensen-Mercer's inequality. Fractal sets are useful tools for describing the accuracy of inequalities in convex functions. The purpose of this paper is to establish a generalized Jensen–Mercer inequality for a generalized convex function on a real linear fractal set  $R_\alpha$  ( $0 < \alpha \leq 1$ ). Further, we also demonstrate some generalized Jensen– Mercer type inequalities by employing local fractional calculus. Lastly, some applications related to Jensen–Mercer inequality and  $\alpha$ -type special means are given. The present approach is efficient, reliable, and may motivate further research in this area.

## Hermite–Hadamard–Mercer and Related Inequalities on Fractal Sets

Saad Ihsan BUTT<sup>1</sup> and Saba YOUSAF<sup>1</sup>

<sup>1</sup>*Department of Mathematics, COMSATS University Islamabad, Lahore Campus, PAKISTAN*

[saadihsanbutt@gmail.com](mailto:saadihsanbutt@gmail.com)

[sabayousafsh9@gmail.com](mailto:sabayousafsh9@gmail.com)

### ABSTRACT

Fractal analysis is a totally new area of research based on local fractional calculus. It has interesting applications in various fields such as a complex graph, computer graphics, the music industry, and picture compression, and many more fields. In the paper, we present new variants of Hadamard–Mercer type inequalities on fractal sets  $R_\alpha$  ( $0 < \alpha \leq 1$ ) by employing generalized convex function. We establish two new lemmas involving local fractional integrals. By using these lemmas, we obtain several results related to generalized Hadamard–Mercer type integral inequalities for local differentiable generalized convex functions on real linear fractal space. Finally, we give applications for probability density functions and compute new generalized means.

## Existence Results for Fractional Differential Equations with Generalized Boundary Conditions

Imran TALIB<sup>1</sup>

<sup>1</sup>*Department of Mathematics, Virtual University of Pakistan, Lahore-PAKISTAN*

[imrantaalib@gmail.com](mailto:imrantaalib@gmail.com)

### ABSTRACT

The presented talk deals with the existence of solutions of nonlinear fractional boundary value problems. The methodology we employ is based on the lower and upper solutions approach along with classical fixed point theorems. The presented approach unifies the existence criteria of various fractional boundary value problems that have been previously dealt separately in the literature. In addition, the Caputo fractional derivative operator is studied at the extreme points. An example is taken to check the suitability of the presented results.

## The Unified Method for The Nonlinear Partial Differential Equations

Md. Nur ALAM<sup>1</sup>

<sup>1</sup>*Department of Mathematics, Pabna University of Science & Technology,  
Pabna-6600, BANGLADESH*

[nuralam23@pust.ac.bd](mailto:nuralam23@pust.ac.bd)  
[nuralam.pstu23@gmail.com](mailto:nuralam.pstu23@gmail.com)

### ABSTRACT

The unified method is a useful method that has appeared in recent times for finding exact solutions of nonlinear partial differential equations (NLPDEs). New obtained exact solutions are different types of soliton wave properties along with trigonometric, hyperbolic, and rational functions solutions. The gained distinguished varieties of exact solutions contain vital applications in engineering and physics. With 3D, 2D, density, and contour graphical illustration, mathematical results explicitly exhibit the proposed algorithm's complete honesty and high performance. From the observation of the outcomes acquired, it is noticed that the unified method can generate essential effects in taking the exact solutions of NLPDEs.

### REFERENCES

- [1] B.S. Bardin and E.A. Chekina, Regul. Chaotic Dynam., 24 (2), 127-144 (2019).
- [2] B. Augner, SIAM J. Control Optim., 57 (3), 1818-1844, (2019).
- [3] S. Islam, M.N. Alam, M.F.A. Asad, C. Tunc, Journal of Applied and Computational Mechanics, 7(2), 715-726 (2021).
- [4] A. Das, Comput. Appl. Math., 37(3), 3208-3225 (2018).
- [5] S. Akcagil, T. Aydemir, NTMSCI, 6(1),185–199, (2018).
- [6] H. Ahmad, M.N. Alam, M.A. Rahman, M.F. Alotaibid, M. Omri, Results in Physics, 29, 104785, (2021).

## On the Initial Value Problem for the Nonlinear Fractional Rayleigh-Stokes Equation

Nguyen Hoang LUC<sup>1,2,3</sup>, Do LAN<sup>4</sup>, Donal O'REGAN<sup>5</sup>, Nguyen Anh TUAN<sup>3</sup>, Yong ZHOU<sup>6</sup>

<sup>1</sup> *Department of Mathematics and Computer Science,  
University of Science Ho Chi Minh City, VIETNAM*

<sup>2</sup> *Vietnam National University, Ho Chi Minh City, VIETNAM*

<sup>3</sup> *Division of Applied Mathematics, Thu Dau Mot University,  
Binh Duong Province, VIETNAM*

<sup>4</sup> *Department of Mathematics, Thuyloi University, 175 Tay Son, Dong Da, Hanoi, VIETNAM*

<sup>5</sup> *School of Mathematics, Statistics and Applied Mathematics,  
National University of Ireland, Galway, IRELAND*

<sup>6</sup> *Faculty of Mathematics and Computational Science,  
Xiangtan University, Hunan 411105, CHINA*

[hoangluctt@gmail.com](mailto:hoangluctt@gmail.com)

[dolan@tlu.edu.vn](mailto:dolan@tlu.edu.vn)

[donal.oregan@nuigalway.ie](mailto:donal.oregan@nuigalway.ie)

[nguyenanhtuan@tdmu.edu.vn](mailto:nguyenanhtuan@tdmu.edu.vn)

[yzhou@xtu.edu.cn](mailto:yzhou@xtu.edu.cn)

### ABSTRACT

An initial-boundary value problem for the nonlinear fractional Rayleigh-Stokes equation is studied in two cases, namely when the source term is globally Lipschitz or locally Lipschitz. The time-fractional derivative used in this work is the classical Riemann-Liouville {derivative}. Thanks to the spectral decomposition, a fixed point argument, and some useful function spaces, the global existence and blow-up property for solutions to the problem are investigated.

### REFERENCES

- [1] E. Bazhlekova, B. Jin, R. Lazarov, and Z. Zhou. Numer. Math. 131, 1-31 (2015).
- [2] C. Fetecau, M. Jamil, C. Fetecau, and D. Vieru. Z. Angew. Math. Phys. 60, 921-933 (2009).

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## Refinements of Jensen's inequality for Jackson Nörlund Integrals

Ammara NOSHEEN<sup>1</sup>, Hafsa MEHMOOD<sup>1</sup> and Khuram Ali KHAN<sup>1</sup>

<sup>1</sup>*Department of Mathematics, University of Sargodha, Sargodha 40100, PAKISTAN*

[khuramsms@gmail.com](mailto:khuramsms@gmail.com)

### ABSTRACT

In this work, we would like to share some refinements of Jensen's inequality involving Jackson Nörlund integrals along with its applications to Hermite-Hadamard inequality, mean value theorems and information theory as well.



## Hermite-Hadamard-Fejer Inequality and Related Inequalities via Alpha and Beta Generators

Yeter ERDAŞ<sup>1</sup> and Erdal ÜNLÜYOL<sup>1</sup>

<sup>1</sup>*Department of Mathematics, Faculty of Science and Arts,  
Ordu University, Ordu-TURKEY*

[yetererdass@gmail.com](mailto:yetererdass@gmail.com)  
[erdalunluyol@odu.edu.tr](mailto:erdalunluyol@odu.edu.tr)

### ABSTRACT

In this study, we obtain the Hermite-Hadamard-Fejer Inequality with respect to alpha and beta generators. Then we established new inequalities related to the Hermite-Hadamard-Fejer Inequality via alpha and beta generators.

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### REFERENCES

- [1] M. Grossmann, R. Katz, Non-Newtonian Calculus, Lee Press Pigeon Cove, Massachusetts, 1972.
- [2] U. Kadak, Non-Newtonian Calculus and its Applications, Ph. D. Thesis, Gazi University, 2015.
- [3] E. Ünlüyol, S. Salaş, İ. İşcan, Convex functions and some inequalities in terms of the Non-Newtonian Calculus April 2017, AIP Conference Proceedings 1833(1):020043, DOI,10.1063/1.4981691, (2017).
- [4] E. Ünlüyol, S. Salaş, İ. İşcan, Generalized star-convex functions and some star-inequalities in terms of the Non-Newtonian Calculus, International Conference On Mathematics and Engineering, 10-12 May, 2017, Turkey.
- [5] U. Kadak, Y. Gurefe, A Generalization on Weighted Means and Convex Functions with respect to the Non-Newtonian Calculus, International Journal of Analysis, 1-9 (2016),
- [6] M.Z. Sarıkaya, Stud. Univ. Babeş-Bolyai Math., 57(3), 377–386 (2012).

## Fekete-Szegö Problem Functional Problems For Some Subclasses of Bi-Univalent Functions Defined By Deniz-Özkan Differential Operator

Murat ÇAĞLAR<sup>2</sup>, Erhan DENİZ<sup>1</sup> and Ziya MİNGSAR<sup>1</sup>

<sup>1</sup>*Department of Mathematics, Faculty of Science and Letters,  
Kafkas University, Kars-TURKEY*

<sup>2</sup>*Department of Mathematics, Faculty of Science,  
Erzurum Technical University, Erzurum-TURKEY*

[edeniz36@gmail.com](mailto:edeniz36@gmail.com)  
[geometri-ziya@gmail.com](mailto:geometri-ziya@gmail.com)  
[mcaglar25@gmail.com](mailto:mcaglar25@gmail.com)

### ABSTRACT

In this study, we solve Fekete-Szegö problem for a new subclass  $B_{\Sigma}^m(\lambda, \beta; \varphi)$  of bi-univalent functions in the open unit disk  $U$  defined by Deniz-Özkan differential operator.

### REFERENCES

- [1] E. Deniz and Y. Özkan, Acta. Uni. Apul. 40, 85-95 (2014).
- [2] M. Fekete and G. Szegö, J. London Math. Soc. 8, 85-89 (1933).
- [3] S. Kazımoğlu and E. Deniz, Hacet. J. Math. Stat. 49(5), 1695-1705 (2020).
- [4] P. Zaprawa, Bull. Belg. Math. Soc. 21(1), 169-178 (2014).

## Coefficient Estimates For A Certain Subclass of Bi-Univalent Functions Defined By using Deniz-Özkan Differential Operator

Ziya MİNGSAR<sup>1</sup>, Erhan DENİZ<sup>1</sup> and Sercan KAZIMOĞLU<sup>1</sup>

<sup>1</sup> *Department of Mathematics, Faculty of Science and Letters,  
Kafkas University, Kars-TURKEY*

[geometri-ziya@hotmail.com](mailto:geometri-ziya@hotmail.com)

[edeniz36@gmail.com](mailto:edeniz36@gmail.com)

[srenkzmglu@gmail.com](mailto:srenkzmglu@gmail.com)

### ABSTRACT

In this paper, we investigate a new subclass  $B_{\Sigma}^m(\lambda, \beta; \varphi)$  of bi-univalent functions in the open unit disk  $U$  defined by Deniz-Özkan differential operator. We obtain initial coefficients bounds.

### REFERENCES

- [1] S. Bulut, Journal of Function Spaces and Applications, Article ID 181932, 7 pages (2013).
- [2] E. Deniz, J. Classical Anal. 2(1), 49-60 (2013).
- [3] E. Deniz and Y. Özkan, Acta. Uni. Apul. 40, 85-95 (2014).
- [4] S. Kazımoğlu and N. Mustafa, Palestine J. Math. 9(2), 1020-1031 (2020).

## **Emergence and Annihilation of Persistent Activity states in Two Population Neural Field Model for under influence Gaussian External Input**

**Muhammad Yousaf BHATTI<sup>1</sup>**

<sup>1</sup>*COMSATS University Islamabad, Lahore Campus, PAKISTAN*

### **ABSTRACT**

In computational neuroscience the Wilson Cowan type two-population neural field model describes the dynamics of interactions between populations of excitatory and inhibitory model neurons [1]. The stationary and symmetric solution of the model called bumps which are Stimulus-specific persistent neural activity is the neural process underlying active (working) memory [2]. Therefore, it is important to investigate the emergence and annihilation of these activities (Bumps). In this work, we have explored effect of Gaussian spatio-temporal external input on the emergence and annihilation of bumps in a two-population neural field model. The external input is divided into three parts, amplitude, spatial part and the temporal part. The effect all these parts of the external input are investigated with focus on temporal part. The emergence and annihilation of the persistent activity states under the influence of triangular spatio-temporal external input is investigated by Yousaf et. al. [2]. The Gaussian temporal function in the external input is closer to natural phenomenon as observed in Roth et. al. [4]. Results also show that the present choice of spatio-external input is better one as compare to the triangular one. It is also found that the relative inhibition time also plays a key role on the emergence and annihilation of the activity.

### **REFERENCES**

- [1] H.R. Wilson and J.D. Cowan, *Biophys. J.*, 12, 1-24 (1972).
- [2] X.J. Wang, *Trends in Neurosciences*, 24, 455-463 (2001).
- [3] M. Yousaf, B. Kriener, J. Wyller, G. T. Einevoll, *Neural Networks*, 46, 75-90 (2013).
- [4] M. Roth, J. C. Dahmen, D. R. Muir, F. Imhof, F. Martini, S. Hofer, *Nature Neurosci.*, 19, 229-307 (2016).

## Piecewise Derivatives Versus Short Memory Concept: Analysis and Application

Abdon ATANGANA<sup>1,2</sup> and Seda İĞRET ARAZ<sup>1,3</sup>

<sup>1</sup>Faculty of Natural and Agricultural Sciences, University of the Free State, SOUTH AFRICA

<sup>2</sup>Department of Medical Research, China Medical University, TAIWAN

<sup>3</sup>Department of Mathematic Education, Faculty of Education,  
Siirt University, Siirt-TURKEY

[AtanganaA@ufs.ac.za](mailto:AtanganaA@ufs.ac.za)  
[sedaaraz@siirt.edu.tr](mailto:sedaaraz@siirt.edu.tr)

### ABSTRACT

We have provided a detailed analysis to show the fundamental difference between the concept of short memory and piecewise differential and integral operators. While the concept of short memory leads to different long tails in different intervals of time or space as results of power law with different fractional orders, the concept of piecewise helps to depict crossover behaviours of different patterns. We presented some examples with different numerical simulations. In some cases, models with piecewise led to crossover behaviours from deterministic to stochastics which is indeed the reason this concept was introduced.

### REFERENCES

- [1] M. Caputo, Geophysical Journal International, 13(5), 529-539 (1967).
- [2] T.R. Prabhakar, Yokohama Math. J., 19, 171-183 (1971).
- [3] M. Caputo, M. Fabrizio, Progress in Fractional Differentiation and Applications, 1(2), 73-85 (2015).
- [4] A. Atangana, D. Baleanu, Thermal Science, 20 (2), 763-769 (2016).
- [5] J. Sabatier, Fractal and Fractional, 4(3), 40 (2020).
- [6] G.C. Wu, Z.G. Deng, D. Baleanu, D.Q. Zeng, Fractional Calculus and Applied Analysis, 22(1), 180-192 (2019).
- [7] A. Atangana, S. Igret Araz, Chaos, Solitons & Fractals, 145, 110638 (2021).
- [8] A. Atangana, S. Igret Araz, Advanced analysis in epidemiological modeling: Detection of wave, 2021, medRxiv.
- [9] B. Ghanbari, D. Kumar, Chaos: An Interdisciplinary Journal of Nonlinear Science, 29(6), 063-103 (2019).
- [10] G. Qi, G. Chen, M.A. Van Wyk, B.J. Van Wyk, Y. Zhang, Chaos Solitons and Fractals, 38, 705-721 (2008).
- [11] G. Qi, Z. Wang Y. Guo, International Journal of Bifurcation and Chaos, 22 (12), 2012.
- [12] T. Mekkaoui, A. Atangana, The European Physical Journal Plus, 132, Article Number 444 (2017).
- [13] A. Atangana S. Igret Araz, *New numerical scheme with Newton polynomial: Theory, Methods and Applications*, Academic Press, Elsevier, 2021, ISBN:9780323854481.

## New Hadamard Type Integral Inequalities via Caputo-Fabrizio Fractional Operators

Ahmet Ocak AKDEMİR<sup>1</sup>, Sinan ASLAN<sup>2</sup>, Merve Nur ÇAKALOĞLU<sup>2</sup> and Erhan SET<sup>3</sup>

<sup>1</sup>*Department of Mathematics, Faculty of Science and Arts,  
Ağrı İbrahim Çeçen University, Ağrı-TURKEY*

<sup>2</sup>*Ağrı İbrahim Çeçen University,  
Institute of Graduate Studies, Ağrı-TURKEY*

<sup>3</sup>*Department of Mathematics, Faculty of Science and Arts,  
Ordu University, Ordu-TURKEY*

[aocakakdemir@gmail.com](mailto:aocakakdemir@gmail.com)

[sinanaslan0407@gmail.com](mailto:sinanaslan0407@gmail.com)

[merve.nur.cakaloglu@gmail.com](mailto:merve.nur.cakaloglu@gmail.com)

[erhanset@yahoo.com](mailto:erhanset@yahoo.com)

### ABSTRACT

In this note, we recalled several new variants of fractional integrals which have become a very popular topic in recent years and have been studied by many mathematicians. Firstly, we have given some new concepts and definitions that have an important role in the development of fractional analysis. In chapter 4, we proved new Hermite-Hadamard type integral inequalities obtained with the help of Caputo-Fabrizio fractional integral operators for  $\alpha_1$ -star  $s$ -convex functions.

- [1] Rashid, S., Safdar, F., Akdemir, A. O., Noor, M. A., & Noor, K. I. (2019). Some new fractional integral inequalities for exponentially  $m$ -convex functions via extended generalized Mittag-Leffler function. *Journal of Inequalities and Applications*, 2019(1), 1-17.
- [2] Nie, D., Rashid, S., Akdemir, A. O., Baleanu, D., & Liu, J. B. (2019). On some new weighted inequalities for differentiable exponentially convex and exponentially quasi-convex functions with applications. *Mathematics*, 7(8), 727.
- [3] Rashid, S., Noor, M. A., Noor, K. I., & Akdemir, A. O. (2019). Some new generalizations for exponentially  $s$ -convex functions and inequalities via fractional operators. *Fractal and Fractional*, 3(2), 24.
- [4] Yaldiz, H., & Akdemir, A. O. (2018). Katugampola fractional integrals within the class of convex functions. *Turkish Journal of Science*, 3(1), 40-50.
- [5] Rashid, S., Akdemir, A. O., Noor, M. A., & Noor, K. I. (2019, August). Generalization of inequalities analogous to preinvex functions via extended generalized Mittag-Leffler functions. In *2019 International Conference on Applied and Engineering Mathematics (ICAEM)* (pp. 256-263). IEEE.
- [6] Li, J. F., Rashid, S., Liu, J. B., Akdemir, A. O., & Safdar, F. (2020). Inequalities involving conformable approach for exponentially convex functions and their applications. *Journal of Function Spaces*, 2020.
- [7] Mumcu, I., Set, E., & Akdemir, A. O. (2019). Hermite-Hadamard type inequalities for harmonically convex functions via Katugampola fractional integrals. *Miskolc Mathematical Notes*, 20(1), 409-424.

## Some New Results for Different Kinds of Convex Functions Caputo-Fabrizio Fractional Operators

Ahmet Ocak AKDEMİR<sup>1</sup>, Sinan ASLAN<sup>2</sup>, Merve Nur ÇAKALOĞLU<sup>2</sup> and Alper EKİNCİ<sup>3</sup>

<sup>1</sup>*Department of Mathematics, Faculty of Science and Arts,  
Ağrı İbrahim Çeçen University, Ağrı-TURKEY*

<sup>2</sup>*Ağrı İbrahim Çeçen University,  
Institute of Graduate Studies, Ağrı-TURKEY*

<sup>3</sup>*Bandırma 17 Eylül University,  
Bandırma Vocational High School, Bandırma-TURKEY*

[aocakakdemir@gmail.com](mailto:aocakakdemir@gmail.com)

[sinanaslan0407@gmail.com](mailto:sinanaslan0407@gmail.com)

[merve.nur.cakaloglu@gmail.com](mailto:merve.nur.cakaloglu@gmail.com)

[alperekinci@hotmail.com](mailto:alperekinci@hotmail.com)

### ABSTRACT

In this paper, we present several general versions of fractional integrals which have become a very popular topic in recent years and have been studied by many mathematicians. First, we present a few definitions and concepts that have an important role in the development of fractional analysis. Finally, we will prove some new Hermite-Hadamard type integral inequalities obtained with the help of Caputo-Fabrizio fractional integral operators.

- [1] Set, E., Akdemir, A. O., & Özdemir, E. M. (2017). Simpson type integral inequalities for convex functions via Riemann-Liouville integrals. *Filomat*, 31(14), 4415-4420.
- [2] Rashid, S., Safdar, F., Akdemir, A. O., Noor, M. A., & Noor, K. I. (2019). Some new fractional integral inequalities for exponentially m-convex functions via extended generalized Mittag-Leffler function. *Journal of Inequalities and Applications*, 2019(1), 1-17.
- [3] Özdemir, M. E., Yıldız, Ç., Akdemir, A. O., & Set, E. (2013). On some inequalities for s-convex functions and applications. *Journal of Inequalities and Applications*, 2013(1), 1-11.
- [4] Nie, D., Rashid, S., Akdemir, A. O., Baleanu, D., & Liu, J. B. (2019). On some new weighted inequalities for differentiable exponentially convex and exponentially quasi-convex functions with applications. *Mathematics*, 7(8), 727.
- [5] Rashid, S., Noor, M. A., Noor, K. I., & Akdemir, A. O. (2019). Some new generalizations for exponentially s-convex functions and inequalities via fractional operators. *Fractal and Fractional*, 3(2), 24.
- [6] Zhou, S. S., Rashid, S., Parveen, S., Akdemir, A. O., & Hammouch, Z. (2021). New computations for extended weighted functionals within the Hilfer generalized proportional fractional integral operators. *AIMS Mathematics*, 6(5), 4507-4525.
- [7] Butt, S. I., Nadeem, M., Qaisar, S., Akdemir, A. O., & Abdeljawad, T. (2020). Hermite-Jensen-Mercer type inequalities for conformable integrals and related results. *Advances in Difference Equations*, 2020(1), 1-24.

Some New Results for Different Kinds of Convex Functions Caputo-Fabrizio Fractional Operators

Ahmet Ocak AKDEMİR, Sinan ASLAN, Merve Nur ÇAKALOĞLU and Alper EKİNCİ- Oral Presentation / 084

## New Integral Inequalities of Hadamard Type via Katugampola Fractional Operator for $m$ –Convex Functions

Merve Nur ÇAKALOĞLU<sup>1</sup>, Ahmet Ocak AKDEMİR<sup>2</sup>, Erhan SET<sup>3</sup> and Sinan ASLAN<sup>1</sup>

<sup>1</sup>*Ağrı İbrahim Çeçen University,*

*Institute of Graduate Studies, Ağrı-TURKEY*

<sup>2</sup>*Department of Mathematics, Faculty of Science and Arts,*

*Ağrı İbrahim Çeçen University, Ağrı-TURKEY*

<sup>3</sup>*Department of Mathematics, Faculty of Science and Arts,*

*Ordu University, Ordu-TURKEY*

[merve.nur.cakaloglu@gmail.com](mailto:merve.nur.cakaloglu@gmail.com)

[aocakakdemir@gmail.com](mailto:aocakakdemir@gmail.com)

[erhanset@yahoo.com](mailto:erhanset@yahoo.com)

[sinanaslan0407@gmail.com](mailto:sinanaslan0407@gmail.com)

### ABSTRACT

The main motivation of this study is to present new HermiteHadamard (HH) type inequalities via a certain fractional operators. We have used an integral identity and give new estimations of HH- type inequalities for differentiable and  $m$  –convex mapping via Katugampola-fractional operators. Main findings of this study would provide elegant connections and general variants of well known results established recently. These results can be extended to different kinds of convex functions as well as pre-invex functions.

- [1] Set, E., Akdemir, A. O., & Özdemir, E. M. (2017). Simpson type integral inequalities for convex functions via Riemann-Liouville integrals. *Filomat*, 31(14), 4415-4420.
- [2] Rashid, S., Safdar, F., Akdemir, A. O., Noor, M. A., & Noor, K. I. (2019). Some new fractional integral inequalities for exponentially  $m$ -convex functions via extended generalized Mittag-Leffler function. *Journal of Inequalities and Applications*, 2019(1), 1-17.
- [3] Özdemir, M. E., Yıldız, Ç., Akdemir, A. O., & Set, E. (2013). On some inequalities for  $s$ -convex functions and applications. *Journal of Inequalities and Applications*, 2013(1), 1-11.
- [4] Nie, D., Rashid, S., Akdemir, A. O., Baleanu, D., & Liu, J. B. (2019). On some new weighted inequalities for differentiable exponentially convex and exponentially quasi-convex functions with applications. *Mathematics*, 7(8), 727.
- [5] Rashid, S., Noor, M. A., Noor, K. I., & Akdemir, A. O. (2019). Some new generalizations for exponentially  $s$ -convex functions and inequalities via fractional operators. *Fractal and Fractional*, 3(2), 24.
- [6] Zhou, S. S., Rashid, S., Parveen, S., Akdemir, A. O., & Hammouch, Z. (2021). New computations for extended weighted functionals within the Hilfer generalized proportional fractional integral operators. *AIMS Mathematics*, 6(5), 4507-4525.
- [7] Butt, S. I., Nadeem, M., Qaisar, S., Akdemir, A. O., & Abdeljawad, T. (2020). Hermite–Jensen–Mercer type inequalities for conformable integrals and related results. *Advances in Difference Equations*, 2020(1), 1-24.

New Integral Inequalities of Hadamard Type via Katugampola Fractional Operator for  $m$ -Convex Functions

Merve Nur ÇAKALOĞLU, Ahmet Ocak AKDEMİR, Erhan SET and Sinan ASLAN– Oral Presentation / 085



## On the Diophantine Equations Involving Integer Sequences

Abdullah ÇAĞMAN<sup>1</sup>

<sup>1</sup>*Department of Mathematics, Faculty of Science and Arts,  
Ağrı İbrahim Çeçen University, Ağrı-TURKEY*

[acagman@agri.edu.tr](mailto:acagman@agri.edu.tr)

### ABSTRACT

In this work, we state all solutions of some Diophantine equations involving Fibonacci, Lucas and Pell sequences via Mathematica Script and prove that there is no other solutions. In our proof, we make use of linear forms in logarithms and Baker-Davenport reduction procedure.

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### REFERENCES

- [1] J. J. Bravo, F. Luca. *Publ. Math. Debrecen*, 82(3-4), 623–639 (2013).
- [2] Y. Bugeaud, M. Mignotte, S. Siksek, *Annals of Mathematics*, 969–1018 (2006).
- [3] A. Dujella, A. Petho, *The Quarterly Journal of Mathematics*, 49(195),291–306 (1998).
- [4] E. M. Matveev. *ii. Izvestiya: Mathematics*, 64(6), 12-17 (2000).
- [5] Z. Siar, F. Erduvan, R. Keskin, *Acta Mathematica Universitatis Comenianae*, 88(2), 247–256 (2019).
- [6] N.P Smart, *The algorithmic resolution of Diophantine equations: a computational cookbook*, volume 41, Cambridge University Press, 1998.

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## Čebyšev's Inequalities

Zlatko PAVIĆ

*Department of Mathematics*

*Mechanical Engineering Faculty, University of Slavonski Brod, Slavonski Brod-CROATIA*

[zpavic@unisb.hr](mailto:zpavic@unisb.hr)

### ABSTRACT

The aim of this presentation is to show the sum and integral form of Čebyšev's inequality, as well as some generalizations. We study Čebyšev's inequalities for ordered  $n$ -tuples and ordered functions. The study offers the concept of proportionally symmetric functions as a generalization of the notion of midpoint symmetric functions. This is the concept that gives the opportunity for expansion of Čebyšev's integral inequality and related inequalities.

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### REFERENCES

- [1] V. I. Levin, S. B. Stečkin, Inequalities, Amer. Math. Soc. Transl. 14, 1-29 (1960).
- [2] A. Matković, J. Pečarić, On a variant of Čebyšev's inequality of the Mercer type, J. Inequal. Appl. 2020, Article 242 (2020).
- [3] A. Witkowski, Inequalities of Levin-Stečkin, Clausing and Čebyšev revisited, arXiv:1612.05373 [math.CA], 1-4 (2016).