SPECTRA AND FINE SPECTRA OF THE GENERALIZED UPPER DIFFERENCE OPERATOR WITH TRIPLE REPETITION Δ_3^{ab} ON THE HAHN SEQUENCE SPACE

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ABSTRACT. The goal of this paper is to obtain the spectra and fine spectra of the matrix Δ_3^{ab} on the Hahn space. Also, we explore some ideas of how to study the problem for a general form of the matrix, namely, the matrix Δ_n^{ab} where the non-zero diagonals are the entries of a n-ary repetition sequence.

1. Introduction

The generalized upper difference operator with triple repetition Δ_3^{ab} is defined on a Banach sequence space by

$$\Delta_3^{ab}x = \Delta_3^{ab}(x_n) = (a_nx_n + b_nx_{n+1})_{n=0}^{\infty} \tag{1.1}$$

where $a_x = a_y$, $b_x = b_y$ for $x \equiv y \,(\mathrm{mod}\ 3)$. The class of the operator Δ_3^{ab} includes, in particular, the operator $U(r,s)$ when $a_k = r$ and $b_k = s$ for all $k \in \mathbb{N}$, with $r, s \in \mathbb{R}$ and $s \neq 0$. Also, it includes the upper difference operator; $a_k = 1$ and $b_k = -1$ for all $k \in \mathbb{N}$. These operators have been studied in [25] and [19], respectively.

Hahn [24] introduced the space h of all sequences $x = (x_k) \in c_0$ such that $\sum\limits_{k=0}^{\infty}(k+1)|x_{k+2} - x_{k+1}|$ is finite. The norm $\|x\|_h = \sum\limits_{k=0}^{\infty}k|x_{k+1} - x_k| + \sup\limits_k|x_k|$ was defined on the space h by Hahn [24]. Rao [34: Proposition 2.1] defined a new norm on h by $\|x\|_h = \sum\limits_{k=1}^{\infty}k|x_{k+1} - x_k|$. The dual space of h is norm isomorphic to the Banach space

$$\sigma_{\infty} = \left\{x = (x_k) \in w : \sup_n \frac{1}{n}\left|\sum_{k=1}^{n}x_k\right| < \infty\right\}.$$

Spectral theory is one of the most useful tools in science. It has many applications in mathematics and physics which contain matrix theory, control theory, function theory, differential and