

DOI: 10.1515/ms-2022-0049 Math. Slovaca **72** (2022), No. 3, 719–736

## SPECTRA AND FINE SPECTRA OF THE GENERALIZED UPPER DIFFERENCE OPERATOR WITH TRIPLE REPETITION $\Delta_3^{ab}$ ON THE HAHN SEQUENCE SPACE

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(Communicated by Gregor Dolinar)

ABSTRACT. The goal of this paper is to obtain the spectra and fine spectra of the matrix  $\Delta_3^{ab}$  on the Hahn space. Also, we explore some ideas of how to study the problem for a general form of the matrix, namely, the matrix  $\Delta_n^{ab}$  where the non-zero diagonals are the entries of a *n*-ary repetition sequence.

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## 1. Introduction

The generalized upper difference operator with triple repetition  $\Delta_3^{ab}$  is defined on a Banach sequence space by

$$\Delta_3^{ab} x = \Delta_3^{ab} (x_n) = (a_n x_n + b_n x_{n+1})_{n=0}^{\infty}$$
(1.1)

where  $a_x = a_y$ ,  $b_x = b_y$  for  $x \equiv y \pmod{3}$ . The class of the operator  $\Delta_3^{ab}$  includes, in particular, the operator U(r,s) when  $a_k = r$  and  $b_k = s$  for all  $k \in \mathbb{N}$ , with  $r,s \in \mathbb{R}$  and  $s \neq 0$ . Also, it includes the upper difference operator;  $a_k = 1$  and  $b_k = -1$  for all  $k \in \mathbb{N}$ . These operators have been studied in [25] and [19], respectively.

Hahn [24] introduced the space h of all sequences  $x = (x_k) \in c_0$  such that  $\sum_{k=0}^{\infty} (k+1) |x_{k+2} - x_{k+1}|$ 

is finite. The norm  $||x||_h = \sum_{k=0}^{\infty} k |x_{k+1} - x_k| + \sup_k |x_k|$  was defined on the space h by Hahn [24].

Rao [34: Proposition 2.1] defined a new norm on h by  $||x||_h = \sum_{k=1}^{\infty} k |x_{k+1} - x_k|$ . The dual space of h is norm isomorphic to the Banach space

$$\sigma_{\infty} = \left\{ x = (x_k) \in w : \sup_{n} \frac{1}{n} \left| \sum_{k=1}^{n} x_k \right| < \infty \right\}.$$

Spectral theory is one of the most useful tools in science. It has many applications in mathematics and physics which contain matrix theory, control theory, function theory, differential and

<sup>2020</sup> Mathematics Subject Classification: Primary 47A10, 47B37.

Keywords: upper triangular band matrix, spectrum, fine spectrum, approximate point spectrum.

This study will be included in PhD Thesis of Rabia Kılıç.

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