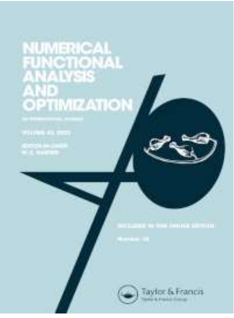

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Inverse Nodal Problems for Dirac-Type Integro-Differential Operators with Linear Functions in the Boundary Condition

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ABSTRACT
In this article, Dirac-type integro-differential operator with linear functions in the boundary condition is considered. We obtain asymptotic expressions for the solution of the differential system and derive the large eigenvalues and nodal points. We also give a constructive procedure for solving an inverse nodal problem. We prove that a dense subset of the nodes determines the coefficients of the differential part of the operator and gives partial information for the integral part of it.

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1. Introduction

The article aims to solve the inverse nodal problem for the boundary value problem (BVP) generated by the following Dirac-type integro-differential system

$$BY'(x) + \Omega(x)Y(x) + \int_0^x M(x,t)Y(t)dt = \mu Y(x),\ \mathrm{x} \in [0,\pi], \tag{1}$$

with the boundary conditions

$$(a_1 + \mu\cos\phi)y_1(0) + (a_2 + \mu\sin\phi)y_2(0) = 0, \tag{2}$$

$$(b_1 + \mu\cos\rho)y_1(\pi) + (b_2 + \mu\sin\rho)y_2(\pi) = 0. \tag{3}$$

Here ϕ, ρ, a_1, a_2, b_1, and b_2 are real constants, $0 \leq\ < \phi, \rho < \pi$, and μ are the spectral parameters,

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$B = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, $\Omega(x) = \begin{pmatrix} p(x) & 0 \\ 0 & q(x) \end{pmatrix}$, $M(x,t) = \begin{pmatrix} M_{11}(x,t) & M_{12}(x,t) \\ M_{21}(x,t) & M_{22}(x,t) \end{pmatrix}$, $Y(x) = \begin{pmatrix} y_1(x) \\ y_2(x) \end{pmatrix}$, $p(x), q(x)$, and $M_{ij}(x,t)$, $(i,j = 1,2)$, are real-valued functions in $W_2^1[0,\pi]$ with respect to t and in $L_2[0,\pi]$ with respect to x.

Inverse spectral problems consist in recovering operators from their spectral characteristics. One of the important spectral characteristics is the zeros of eigenfunctions called nodes or nodal points. The inverse nodal problems for Sturm–Liouville operator was first handled and resolved by McLaughlin in 1988 [1]. In this study, it has been shown that a dense subset of zeros of eigenfunctions uniquely determines the potential of the Sturm Liouville operator up to the mean value. In 1989, Hald and McLaughlin gave a numerical method for reconstructing potential function from nodes for more general boundary conditions [2]. In 1997, Yang suggested a constructive procedure for reconstructing the potential and the boundary condition of the Sturm–Liouville problem from nodes of its eigenfunctions [3]. In this work, the author also gave the uniqueness theorem for general boundary conditions using the same method as McLaughlin. Inverse nodal problems have been addressed by various researchers in several papers for different operators [4–14]. The inverse nodal problems for Dirac operators with various boundary conditions have been studied and shown that a dense subset of nodes is enough to determine the coefficients of the operators in [15–17].

In recent years, integro-differential operators attracted much attention of mathematicians. Such operators have important applications in many fields of science (see monographs [18, 19] and references therein). Therefore, many researchers are currently working on inverse problems for these operators [20–35]. The inverse nodal problem for Dirac-type integro-differential operators with Robin boundary conditions was first studied by [36]. This operator with parameter-dependent boundary conditions linearly and nonlinearly were studied by [37] and [38], respectively. In their studies, the authors considered $p(x)$ and $q(x)$, which are the components of the potential function $\Omega(x)$, as a special case such that $p(x) - q(x) = const$. In this study, we consider $p(x)$ and $q(x)$ as two independent functions and investigate a more general case. In this way, we have the opportunity to determine $p(x)$ and $q(x)$ separately. We deal with an inverse nodal problem of reconstructing the Dirac-type integro-differential operators with the spectral parameter contained linearly in the both boundary conditions. It is shown that the coefficients of the differential part of the operator can be determined using a dense subset of the nodes. For this problem, we also give a