

SPECTRA AND FINE SPECTRA OF THE GENERALIZED UPPER DIFFERENCE OPERATOR WITH TRIPLE REPETITION Δ_3^{ab} ON THE HAHN SEQUENCE SPACE

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ABSTRACT. The goal of this paper is to obtain the spectra and fine spectra of the matrix Δ_3^{ab} on the Hahn space. Also, we explore some ideas of how to study the problem for a general form of the matrix, namely, the matrix Δ_n^{ab} where the non-zero diagonals are the entries of a n -ary repetition sequence.

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1. Introduction

The generalized upper difference operator with triple repetition Δ_3^{ab} is defined on a Banach sequence space by

$$\Delta_3^{ab}x = \Delta_3^{ab}(x_n) = (a_n x_n + b_n x_{n+1})_{n=0}^\infty \tag{1.1}$$

where $a_x = a_y, b_x = b_y$ for $x \equiv y \pmod{3}$. The class of the operator Δ_3^{ab} includes, in particular, the operator $U(r, s)$ when $a_k = r$ and $b_k = s$ for all $k \in \mathbb{N}$, with $r, s \in \mathbb{R}$ and $s \neq 0$. Also, it includes the upper difference operator; $a_k = 1$ and $b_k = -1$ for all $k \in \mathbb{N}$. These operators have been studied in [25] and [19], respectively.

Hahn [24] introduced the space h of all sequences $x = (x_k) \in c_0$ such that $\sum_{k=0}^\infty (k+1) |x_{k+2} - x_{k+1}|$ is finite. The norm $\|x\|_h = \sum_{k=0}^\infty k |x_{k+1} - x_k| + \sup_k |x_k|$ was defined on the space h by Hahn [24].

Rao [34: Proposition 2.1] defined a new norm on h by $\|x\|_h = \sum_{k=1}^\infty k |x_{k+1} - x_k|$. The dual space of h is norm isomorphic to the Banach space

$$\sigma_\infty = \left\{ x = (x_k) \in w : \sup_n \frac{1}{n} \left| \sum_{k=1}^n x_k \right| < \infty \right\}.$$

Spectral theory is one of the most useful tools in science. It has many applications in mathematics and physics which contain matrix theory, control theory, function theory, differential and

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This study will be included in PhD Thesis of Rabia Kiliç.

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