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Approximations of the Images and Integral Funnels of the L_p Balls under a Urysohn-Type Integral Operator

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Abstract

Approximations of the image and integral funnel of a closed ball of the space L_p , $p > 1$, under a Urysohn-type integral operator are considered. A closed ball of the space L_p , $p > 1$, is replaced by a set consisting of a finite number of piecewise constant functions, and it is proved that, for appropriate discretization parameters, the images of these piecewise constant functions form an internal approximation of the image of the closed ball. This result is applied to approximate the integral funnel of a closed ball of the space L_p , $p > 1$, under a Urysohn-type integral operator by a set consisting of a finite number of points.

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