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# On the Continuity Properties of the Set of Trajectories of the Control System with Limited Control Resources

Anar Huseyin[1]*

[1] Sivas Cumhuriyet University, Faculty of Science, Department of Statistics and Computer Sciences, Sivas, Türkiye
*ahuseyin@cumhuriyet.edu.tr
*Orcid No: 0000-0002-3911-2304

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**Abstract**

In this paper the control system with integral constraint on the control functions is studied where the behavior of the system by the Urysohn type integral equation is described. The admissible control functions are chosen from the closed ball of the space $L_p([a,b];R^m)$ $(p>1)$ centered at the origin with radius $r$. Dependence of the set of trajectories on $r$ and $p$ is investigated. It is proved that the set of trajectories is Lipschitz continuous with respect to $r$ and is continuous with respect to $p$. The robustness of the trajectory with respect to the fast consumption of the remaining control resource is established.

**Keywords:** control system, Hausdorff distance, integral constraint, robustness, Urysohn integral equation.

## 1. Introduction

The control systems arise in different areas of physics, mechanics, airspace navigation, economics, sociology, etc. and depending on character of control efforts are classified as control systems with geometric constraints, integral constraints and mixed constraints on the control functions. The theory of control systems with geometric constraints on the control functions is enough well investigated chapter of the control systems theory (see, e.g. [4], [13], [16], [20] and references therein). But integral constraints on the control functions arise in the cases when the control resource is exhausted by consumption such as energy, fuel, finance, etc. (see, e.g. [3], [6], [12], [15], [18], [21], [22], [23]). Note that integral boundedness of the control function does not imply its geometric boundedness. This situation causes additional difficulties and therefore studying the control systems with integral constraints on the control functions requires special methods.

Integral equations are very adequate tool to describe the behaviors of various processes arising in the theory and applications (see, e.g. [2], [7], [17], [19], [24]). In this paper the control system described by Uryshon type integral equation is considered. The control functions are chosen from the closed ball of the space $L_p([a,b];R^m)$ $(p>1)$ centered at the origin with radius $r$. Note that the different topological properties and approximate constructions methods of the set of trajectories of the control systems described by various type integral equations and integral constraints on the control functions are studied in papers [8-11].

The paper is organized as follows. In Section 2 the basic conditions and propositions are formulated which are used in following arguments. In Section 3 it is proved that the set of trajectories is Lipschitz continuous with respect to $r$ (Theorem 3.1). In Section 4 it is shown that the set of trajectories depends on $p$ continuously (Theorem 4.1). In Section 5 it is proved that system's trajectory is robust with respect to the fast consumption of the remaining control resource (Theorem 5.1) and it is shown that every trajectory can be approximated by trajectory obtained by the full consumption of the available control resource (Theorem 5.2).

## 2. The System's Description

Consider control system the behavior of which is described by Urysohn type integral equation

$$x(t)=f(t,x(t))+\lambda\int_a^b K\big(t,s,x(s),u(s)\big)ds \quad \mathbf{(2.1)}$$

where $t\in[a,b]$, $x(t)\in R^n$ is the state vector, $u(s)\in R^m$ is the control vector, $\lambda\geq 0$.
For given $p>1$ and $r\geq 0$ we denote